KENDRIYA VIDYALAYA SANGATHAN

तत् त्वं पूषन् अपावृणु

केन्द्रीय विद्यालय संगठन

Kendriya Vidyalaya Sangathan's Symbol

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SUBJECT ENRICHMENT MATERIAL

(FOR HIGH ACHIEVERS)

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INDEX

CHAPTER-1 : REAL NUMBERS

- **Euclid's division lemma**
- **Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples**
- **Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$**
- **Decimal representation of rational numbers interms of terminating/non-terminating recurring decimals.**

IMPORTANT POINTS TO REMEMBER

- ❖ **EUCLID'S DIVISION LEMMA**

  (Euclid's Division Lemma) : Given positive integers a and b, there exist unique integers q and r satisfying a = bq + r, $0 \leq r < b$. Here we call 'a' as dividend, 'b' as divisor, 'q' as quotient and 'r' as remainder.

  Dividend = (Divisor x Quotient) + Remainder

  If in Euclid's lemma r = 0 then b would be HCF of 'a' and 'b'.
- ❖ **An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.**
- ❖ **A lemma is a proven statement used for proving another statement**
- ❖ **EUCLID'S DIVISION ALGORITHM**

  Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two given positive integers. Recall that the HCF of two positive integers a and b is the largest positive integer d that divides both a and b.
- ❖ **To obtain the HCF of two positive integers, say c and d, with c > d, follow the steps below:**

  Step 1: Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that c = dq + r, $0 \leq r < d$.

  Step 2 : If r = 0, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.

  Step 3: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

  This algorithm works because HCF (c, d) = HCF (d, r) where the symbol HCF (c, d) denotes the HCF of c and d, etc.
- ❖ **Euclid's division lemma/algorithm has several applications related to finding properties of numbers.**

- **(Fundamental Theorem of Arithmetic): Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.**
  The prime factorisation of a natural number is unique, except for the order of its factors.
  - **Property of HCF and LCM of two positive integers 'a' and 'b':**
    - **HCF (a, b) × LCM (a, b) = a × b**
    - **LCM (a, b) =** $\frac{a \times b}{HCF(a,b)}$
    - **HCF (a, b) =** $\frac{a \times b}{LCM(a,b)}$
- **PRIME FACTORISATION METHOD TO FIND HCF AND LCM**
  - **HCF (a, b) = Product of the smallest power of each common prime factor in the numbers.**
  - **LCM (a, b) = Product of the greatest power of each prime factor, involved in the numbers.**
  - **HCF (p, q, r) × LCM (p, q, r) ≠ p × q× r, where p, q, r are positive integers**
  - **However, the following results hold good for three numbers p, q and r:**
    - **LCM (p, q, r) =** $\frac{p.q.r\ .\ HCF(p,q,r)}{HCF(p,q).\ HCF(q,r).\ HCF(r,p)}$
    - **HCF(p, q , r) =** $\frac{p.q.r\ .\ LCM(p,q,r)}{LCM(p,q).\ LCM(q,r).\ LCM(r,p)}$

IMPORTANT QUESTIONS FROM CHAPTER

1 Find the least number that is divisible by all the numbers from 1 to 10 (both inclusive).

2 If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is
(a) an even number (b) an odd number (c) an odd prime number (d) a prime number

3 A class of 20 boys and 15 girls is divided into n groups so that each group has x boys and y girls. Find x, y and n. what values are referred in a class?

4 Prove that one of any three consecutive positive integers must be divisible by 3.

5 Can we have two numbers with 24 & 888 as their HCF & LCM respectively?

6 For any positive integer n, prove that $n^3 - n$ is divisible by 6.

7 The number $3^{13} - 3^{10}$ is divisible by
(a) 2 and 3 (b) 3 and 10 (c) 2, 3 and 10 (d) 2, 3 and 13

8 Find HCF of 81 and 237 and express it as a linear combination of 81 and 237 i.e. HCF (81, 237) = 81x + 237y for some and y.

9 Show that there is no positive integer n, for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

10 A book seller has 420 science stream books and 130 Arts stream books. He wants to stack them in such a way that each stack has the same number and they take up the least area of the surface.

(i) What is the maximum number of books that can be placed in each stack for this purpose?

(ii) Which mathematical concept is used to solve the problems?

11 Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

12 Two positive numbers have their HCF as 12 and their product as 6336. The number of pairs possible for the numbers, is

(a) 2 (b) 3 (c) 4 (d) 5

13 The value of $(12)^{3^x} + (18)^{3^x}$, $x \in$ N,end with the digit.

(a) 2 (b) 8 (c) 0 (d) Cannot be determined

14 Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together? How many times will they toll together in 20 hours excluding the one at the start?

15 Find HCF and LCM of 16 and 36 by prime factorization and check your answer.

16 The least number which is a perfect square and is divisible by each of 16, 20 and 24 is

(a) 240 (b) 1600 (c) 2400 (d) 3600

17 Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p, q are primes.

18 Show that one and only one out of n, n + 4, n + 8, n + 12 and n + 16 is divisible by 5, where n is any positive integer.

19 If n is an odd integer, then show that $n^2 - 1$ is divisible by 8.

20 Prove that if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

21 Show that the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.

ANSWERS

1 2520

2 (a) an even number

3 HCF(20,15) = 5

So number of groups are 5

x = 4, y = 3 (number of students in each group) = $\frac{20+15}{5} = 7$

i. Co-education is promoted

ii. Dividing class into small groups for better learning and healthy competition.

4 Let a be any positive integer. On dividing a by 3 let q is the quotient and r is the remainder.

So a = bq + r, where $0 \leq r < 3$

So a = bq + r, where r = 0, 1, 2

Three positive integers are a, a + 1, a + 2

Case 1: If r = 0 then a = 3q

So a **= 3q , which is divisible by 3**

a + 1 = 3q + 1, which is not divisible by 3

a + 2 = 3q + 2, which is not divisible by 3

Case 2: If r = 1 then a = 3q + 1

So a = 3q + 1 , which is not divisible by 3

a + 1 = 3q + 2, which is not divisible by 3

a + 2 = 3(q + 1), which is divisible by 3

Case 3: If r = 2 then a = 3q + 2

So a = 3q + 2 , which is not divisible by 3

a + 1 = 3(q +1), which is divisible by 3

a + 2 = 3(q + 1) + 1, which is not divisible by 3

5 Yes as HCF =24 also divides the LCM = 888

6 We have $n^3 - n = n (n^2 - 1) = n (n-1) (n+1)$

Thus $n^3 - n$ is product of three consecutive positive integers.

Since, any positive integers a is of the form 3q, 3q +1, 3q +2 for some integer q.

Let a, a + 1, a + 2 be any three consecutive integers.

Case I: a = 3q

If a = 3q then,

a(a + 1)(a + 2) = 3q(3q +1)(3q +2)

Product of two consecutive integers (3q +1)(3q +2) is an even integer, say 2r.

Thus a(a + 1)(a + 2) = 3q.2r = 6qr, which is divisible by 6.

Case II: a = 3q + 1

If a = 3q + 1 then

a(a + 1)(a + 2) = (3q+1)(3q+2)(3q +3) = (3q+1)(3q+2)(3)(q+1) = (2r) (3)(q+1) = 6r(q+1)

which is divisible by 6.

Case III: a = 3q + 2

If a = 3q + 2 then

a(a + 1)(a + 2) = (3q+2)(3q+3)(3q +4)

= (3q+2)(3q+3)(3q+4)

= (3)(q+1)(3q+2)(3q+4)

= 6r(q+1) which is divisible by 6.

Hence, the product of three consecutive integers is divisible by 6

So $n^3 - n$ is divisible by 6.

7 (d) 2, 3 and 13

$3^{13} - 3^{10} = 3^{10}(3^3 - 1) = 3^{10}(26) = 3^{10}(2 \times 13)$

Hence, The number $3^{13} - 3^{10}$ is divisible by 2, 3 and 13.

8 By using Euclid's Division Lemma, we have

$237 = 81 \times 2 + 75$ ...(1)

$81 = 75 \times 1 + 6$ ...(2)

$75 = 6 \times 12 + 3$ ...(3)

$6 = 3 \times 2 + 0$ ...(4)

Hence, HCF(81, 237) = 3

In order to write 3 in the form of 81x + 237y

$3 = 75 - (6 \times 12) = = 75 - (81 - 75) \times 12)$ Replace 6 from (2)

$= 75 \times 13 - 81 \times 12 = (237 - 81 \times 2) \times 13 - 81 \times 12$ Replace 75 from (1)

$= 237 \times 13 - 81 \times 38 = 81x + 237y$

Hence x = - 38 =– and y =13. These values of x and y are not unique

9 Let us assume that there is a positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational and equal to $\frac{p}{q}$, where p and q are positive integers ,HCF(p, q) = 1 and q$\neq$ 0.

$\sqrt{n-1} + \sqrt{n+1} = \frac{p}{q}$ ....(1)

$\frac{1}{\sqrt{n-1}+\sqrt{n+1}} = \frac{q}{p}$

On rationalizing

$\sqrt{n+1} - \sqrt{n-1} = \frac{2q}{p}$ ....(2)

Adding (1) and (2), we get $\sqrt{n+1} = \frac{p^2+2q^2}{2pq}$

Subtracting (2) from (1) we have $\sqrt{n-1} = \frac{p^2-2q^2}{2pq}$

10 Given number of science books = 420 and number of Arts books = 130

$420 = 2\times2\times3\times5\times7$

$130 = 2\times5\times13$

(i) Maximum number of books that can be placed in each stack for the given purpose = 10

(ii) Prime factorisation method.

11 $n^2 - n = n(n-1)$

Thus $n^2 - n$ is product of two consecutive positive integers.

Any positive integer is of the form 2q or 2q+1, for some integer q.

Case 1 : n = 2q

If n=2q we have

$n^2 - n = n(n-1) = 2q(2q-1) = 2m$ where $m = 2q(2q-1)$ which is divisible by 2.

Case 2 : n = 2q +1

$n^2 - n = n(n-1) = (2q+1)(2q+1-1) = 2q(2q+1) = 2m$ where $m = 2q(2q+1)$ which is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n

12 (a) 2

Let the numbers be 12x and 12y where x and y are co-primes.

Product of these numbers = 144xy

Hence, $xy = \frac{6336}{144} = 44$

Since, 44 can be written as the product of two factors in three ways. i.e. $1\times 44,\ 2\times 22, 4\times 11$

As x and y are co-prime, so (x, y) can be (1,44) or (4,11) but not (2,22).

Hence, two possible pairs exist.

13 (c) 0

The value of $(12)^{3^x} + (18)^{3^x}$, $x \in$ N, end with the digit.

For all $x \in$ N,

$If\ (12)^{3^x}\ ends\ with\ 8, then\ (18)^{3^x}\ ends\ with\ 2.$

$If\ (12)^{3^x}\ ends\ with\ 2, then\ (18)^{3^x}\ ends\ with\ 8.$

Thus $(12)^{3^x} + (18)^{3^x}\ ends\ with\ 0\ only.$

14 The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$9 = 3 \times 3 = 3^2$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 3 \times 5$

LCM (9, 12, 15) = $2^2 \cdot 3^2 . 5^1$ = 150 minutes

The bells will toll next together after 150 minutes.

Three bells toll together in 20 hours = $20 \div 2.5 = 8\ times$

15 $16 = 2 \times 2\times 2\times 2 = 2^4$

$36 = 2\times 2\times 3\times 3 = 2^2 \times 3^2$

HCF(16, 36) $= 2^2 = 4$

LCM (16, 36) $= 2^4 \times 3^2 = 16 \times 9 = 144$

To check HCF and LCM by using formula

HCF(a, b) × LCM(a, b) = a × b

or, 4 × 144 = 16 × 36

576 = 576

Thus LHS = RHS

16 (d) 3600

The L.C.M. of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect number.

17 $Let\ \sqrt{p} + \sqrt{q}$ is rational number.

So $\sqrt{p} + \sqrt{q} = \frac{a}{b}$, where a, b are integers having no common factors other than 1 and b $\neq 0$.

$(\sqrt{p} + \sqrt{q})^2 = (\frac{a}{b})^2$

$p + q + 2\sqrt{pq} = \frac{a^2}{b^2}$

$\sqrt{pq} = (\frac{a^2}{b^2} - p - q) \div 2$

It is contradiction as irrational $\neq rational\ number.$

So $\sqrt{p} + \sqrt{q}$ is irrational number.

18 Any positive integer can be written in the form 5q, 5q+1, 5q+2, 5q+3, 5q+4

Case 1: If n = 5q

n = 5q, which is divisible by 5

n + 4 = 5q + 4, which is not divisible by 5

n + 8 = 5q + 8 = 5(q + 1) + 3, which is not divisible by 5

n + 12 = 5q + 12 = 5(q + 2) + 2, which is not divisible by 5

n + 16 = 5q + 16= 5(q + 3) + 1, which is not divisible by 5

Case 2: If n = 5q + 1

n = 5q + 1, which is not divisible by 5

n + 4 = 5q + 1 + 4 = 5(q + 1), which is divisible by 5

n + 8 = 5q + 1 + 8 = 5(q + 1) + 4, which is not divisible by 5

n + 12 = 5q + 1 + 12 = 5(q + 2) + 3, which is not divisible by 5

n + 16 = 5q + 1 + 16= 5(q + 3) + 2, which is not divisible by 5

Case 3: If n = 5q + 2

n = 5q + 2, which is not divisible by 5

n + 4 = 5q + 2 + 4 = 5(q + 1) + 1, which is not divisible by 5

n + 8 = 5q + 2 + 8 = 5(q + 2), which is divisible by 5

$n + 12 = 5q + 2 + 12 = 5(q + 2) + 4$, which is not divisible by 5

$n + 16 = 5q + 2 + 16= 5(q + 3) + 3$, which is not divisible by 5

Case 4: If $n = 5q + 3$

$n = 5q + 3$, which is not divisible by 5

$n + 4 = 5q + 3 + 4 = 5(q + 1) + 2$, which is not divisible by 5

$n + 8 = 5q + 3 + 8 = 5(q + 2) + 1$, which is not divisible by 5

$n + 12 = 5q + 3 + 12 = 5(q + 3)$, which is divisible by 5

$n + 16 = 5q + 3 + 16= 5(q + 3) + 4$, which is not divisible by 5

Case 5: If $n = 5q + 4$

$n = 5q + 4$, which is not divisible by 5

$n + 4 = 5q + 4 + 4 = 5(q + 1) + 3$, which is not divisible by 5

$n + 8 = 5q + 4 + 8 = 5(q + 2) + 2$, which is not divisible by 5

$n + 12 = 5q + 4 + 12 = 5(q + 3) + 1$, which is not divisible by 5

$n + 16 = 5q + 4 + 16= 5(q + 4)$, which is divisible by 5

19 If n is a odd number then $n = 2a + 1$, where a is any integer.

$n^2 – 1 = (2a +1)^2 – 1 = 4a(a +1)$, which is divisible by 8 as $a(a + 1)$ is also divisible by 2.

20 Let $x = 2a + 1$ and $y = 2a + 3$ where a is any positive integer.

$x^2 + y^2 = (2a + 1)^2 + (2a + 3)^2 = 8(a^2 + 2a + 1) + 2$, which is not divisible by 4

21 Any positive integer n can be written in the form $6q$, $6q + 1$, $6q + 2$, $6q + 3$, $6q + 4$ or $6q + 5$

Case 1: If $n = 6q$

$n^2 = (6q)^2 = 6(6q^2) =$ **$6m$**,where $m = 6q^2$

Case 2: If $n = 6q + 1$

$n^2 = (6q + 1)^2 = 6(6q^2 + 2q) + 1 =$ **$6m + 1$**,where $m = 6q^2 + 2q$

Case 3: If $n = 6q + 2$

$n^2 = (6q + 2)^2 = 6(6q^2 + 4q) + 4 =$ **$6m + 4$**,where $m = 6q^2 + 4q$

Case 4: If $n = 6q + 3$

$n^2 = (6q + 3)^2 = 6(6q^2 + 6q + 1) + 3 =$ **$6m + 3$**,where $m = 6q^2 + 6q + 1$

Case 5: If $n = 6q + 4$

$n^2 = (6q + 4)^2 = 6(6q^2 + 8q + 2) + 4 =$ **$6m + 4$**,where $m = 6q^2 + 8q + 2$

Case 6: If $n = 6q + 5$

$n^2 = (6q + 5)^2 = 6(6q^2 + 10q + 4) + 1 =$ **$6m + 1$**,where $m = 6q^2 + 10q + 4$

This shows that the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m.

Chapter – 2

Polynomials

Level - 1

1. The value of k for which (–4) is a zero of the polynomial $x^2 - x - (2k + 2)$ is
   (a) 3 (b) 9 (c) 6 (d) –1
2. A quadratic polynomial whose zeroes are –3 and 4 is
   a) $x^2 - x + 12$ b) $x^2 + x + 12$ c) $2x^2 + 2x - 24$. d) none of the above.
3. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then value of k is
   (a) 2 (b) 4 (c) –2 (d) – 4
4. Which are the zeroes of $p(x) = x^2 + 3x - 10$:
   (a) 5, –2 (b) –5, 2 (c) –5, –2 (d) none of these
5. If $p(x) = 3x^3 - 2x^2 + 6x - 5$.Find p(2).
6. Find the zeroes of the polynomial $x^2 - 3$
7. Find the zeroes of the polynomial $mx^2 + (m + n)x + n$.

Level – 2

1. If one of the zero of the polynomial $f(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other then k =
   (a) 2 (b) 1 (c) –1 (d) – 2
2. If m and n are zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$
3. If a and b are zeroes of the polynomial $x^2 - x - 6$, then find a quadratic polynomial whose zeroes are (3a + 2b) and (2a + 3b).
4. If 2 and – 3 are the zeroes of the polynomial $x^2 + (a + 1)x + b$, then find the value of a and b.
5. Find the zeroes of quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.
6. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other. Find the value of a.
7. Write a polynomial whose zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

Level - 3

1. If p and q are the zeroes of $p(x) = kx^2 - 3x + 2k$ and p+q=pq then find the value of k.
2. If α, β are the two zeroes of the polynomial $25p^2 - 15p + 2$, find a quadratic polynomial whose zeroes are 1/2α and 1/2β.
3. Obtain all zeroes of polynomial $f(x) = x^4 - 2x^3 - 26x^2 + 54x - 27$ if two of its zeroes are $3\sqrt{3}$ and $-3\sqrt{3}$.
4. If three zeroes of polynomial $x^4 - x^3 - 3x^2 + 3x$ are 0, $\sqrt{3}$ and $-\sqrt{3}$. Find its fourth zero.
5. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be (ax + b), find a and b.

6. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be $px + q$, find the value of p and q.
7. Find all the zeroes of the polynomial $2x^4 - 10x^3 + 5x^2 + 15x - 12$, if it is given that two of its zeroes are $\sqrt{\frac{3}{2}}$ and $-\sqrt{\frac{3}{2}}$.

Answers:

Level – 1

1. b) 2. d) 3. b) 4. b) 5. 23 6. $\sqrt{3}, -\sqrt{3}$ 7. $-\frac{1}{m}, -\frac{1}{n}$

Level – 2

1. a) 2. -145/12 3. x^2-5x 4. a=0 , b= -6 5. $\frac{2}{\sqrt{3}}, 2\sqrt{3}$ 6. a= 3

7. x^2-4x+1

Level – 3

1. k = 3/2 2. $8p^2-30p+25$ 3. $3\sqrt{3}, -3\sqrt{3}, 1, 1$ 4. Fourth zero = 1
5. a = 1, b = 2 6. p = 2, q = 3 7. 4,1

CHAPTER 3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

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1. **At a certain time in a deer park, the number of heads and the number of legs of deer and human visitors were counted and it was found there were 39 heads & 132 legs. Find the number of deer and human visitors in the park.**

2. **Solve for x, y**

   $7(y + 3) 2(x + 2) = 14,$

   $4 (y 2) + 3(x 3) = 2$

3. **Find the value of p and q for which the system of equations represent coincident lines**

   $2x +3y = 7,$

   $(p+q+1)x +(p+2q+2)y =4(p+q)+1$

4. **Students are made to stand in rows. If one student is extra in a row there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of students in the class.**

5. The larger of two supplementary angles exceeds the smaller by 18^0, find them.

6. **A train covered a certain distance at a uniform speed. If the train would have been 6km/hr faster, it would have taken 4 hours less than the scheduled time. And if the train were slower by 6km/hr, it would have taken 6 hours more than the scheduled time. Find the distance of the journey.**

7. **A chemist has one solution which is 50% acid and a second which is 25% acid. How much of each should be mixed to make 10 litres of 40% acid solution.**

8. **In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes & this later number was equal to twice his majority over B. If there were 18000 persons on the electoral roll. How many voted for B.**

9. When the son will be as old as the father today their ages will add up to 126 years. When the father was old as the son is today, their ages add upto 38 years. Find their present ages.

10. **A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30km at half speed in 5 hours. If the breakdown had occurred 10km farther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.**

ANSWERS

Ans: Let the no. of deers be x And no. of humans be y

ATQ:

$x + y = 39$ …….. (1)

$4x + 2y = 132$ ……. (2)

Multiply (1) and (2)

On solving, we get …

$x = 27$ and $y = 12$

∴ No. of deers = 27 and No. of humans = 12

2**Ans:** $7(y + 3) - 2(x + 2) = 14$ ………. (1)

$4(y - 2) + 3(x - 3) = 2$ ………. (2)

From (1) $7y + 21 - 2x - 4 = 14$

On solving, we will get….

$2x - 7y - 3 = 0$ ………. (3)

From (2) $4y - 8 + 3x - 9 = 2$

On solving, we will get….

$3x + 4y - 19 = 0$ ………. (4)

$2x - 7y - 3$

$3x + 4y - 19$

Substitute this, to get $y = 1$ and $x = 5$

∴ $x = 5$ and $y = 1$

∴

3 Ans. $a_2 = p + q + 1$, $b_2 = p + 2q + 2$, $c_2 = (p + q) + 1$

For the following system of equation the condition must be

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\Rightarrow \frac{2}{p+q+1} = \frac{3}{p+q+2} = \frac{7}{4(p+q)+1}$$
$$\Rightarrow \frac{2}{p+q+1} = \frac{7}{4(p+q)+1}$$

7p +14q + 14 = 12p + 12q + 3

5p - 2q - 11 = 0 ………. (2)
p + q + - 5 = 0
5p - 2q - 11 = 0
From (1) and (2)
5p + 5q -25 = 0
5p - 2q - 11 = 0

Solve it, to get q = 2

Substitute value of q in equation (1)

p + q - 5 = 0

On solving we get, p = 3 and q = 2

4 Ans: No. of rows be y

Let the number of students be x

Number of students in the class will be = xy
One student extra, 2 rows less

(x + 1) (y – 2) = xy
xy – 2x + y -2 = xy -(-2x + y -2) = 0
+2x – y = -2 ……….. (1)

One student less, three more rows
(x - 1) (y + 3) = xy

xy + 3x - y -3 = xy

$3x - y = 3$ ....(2)

From (1) & (2)

$2x - y = -2 \times 3$

$3x - y = 3 \times -2$

Solve it, to get ... $y = 12$
and $x = 5$ ∴ Number of
student = xy

$= 12 \times 5$

= 60 students

5 Ans: $\boldsymbol{x + y = 180^0}$

$$\boldsymbol{x - y = 18^0}$$

$2x = 198$

$x = 198 / 2 = x = 99^0$

$x + y = 180$

$y = 180 - 99$

$y = 81^0$

6 Ans: Let the speed of the train by x km/hr And the time taken by it by y

Now distance traveled by it is x x y = xy

APQ:

I--- (x + 6) (y - 4) = xy
4x – 6y = -24
=> 2x – 3y = -12 ……….(1)

II--- (x - 6) (y+ 6) = xy
6x – 6y= 36

=> x– y= 6 ………. (2)

Solving for x and y we get y = 24, x = 30 So the distance =30 × 24 720 km

7 Ans: Let 50 % acids in the solution be x Let 25 % of other solution be y

Total Volume in the mixture = x + y
A.P.Q:
x + y = 10 ………. (1)

A.P.Q:

$$\frac{50}{100}x+\frac{25}{100}y=\frac{40}{100}\times10$$

2x + y = 16 ………. (2)

So x = 6 & y = 4

8 Ans: Let x and y be the no. of votes for A & B respectively.
The no. of persons who did not vote = (18000 – x – y)
APQ:
x = 2(18000 – x – y)

⇒ 3x + 2y = 36000 ………. (1)

&

$(18000 - x - y) = (2)(x - y)$

$\Rightarrow 3x - y = 18000$ .... (2)

On solving we get, $y = 6000$ and $x = 8000$

Vote for B = 6000

9 Ans: let the son's

present age be x Father's

age be y

Difference in age $(y - x)$

Of this difference is added to the present age of son, then son will be as old as the father now and at that time, the father's age will be $[y + (y - x)]$

APQ:

$[x + (y - x)] + [y (y - x)] = 26$
$[y + (x - y)] + [x + (x - y)] = 38$
Solving we get the value of x and y

10 Ans: Let x be the place where breakdown

occurred y be the original speed

On solving, we get, x = 10 km and y = 10 km/h

CHAPTER-4

QUADRATIC EQUATIONS

1.	For what value of c, roots of quadratic equation $4x^2 - 2x + (c - 4) = 0$ are reciprocal of each other. (a) 4 (b) 6 (c) 1 (d) 8	(1)
2.	Find the value of p for which the two roots of the equation : $x^2 – 2(p-2)x + (4p-3) =0$ are equal ? (a) 2 (b) 6 (c) 1 or 7 (d) 3	(1)
3.	If the two roots of a quadratic equation are $\sqrt{2}$ and 1,then quadratic equation will be ----(a) $2x^2 = 1$ (b) $x^2 – (\sqrt{2} + 1)x + \sqrt{2}$ (c) $2x^2 -1$ (d) $x^2 - 2x + 1$	(1)
4.	For what value of p , equation $px^2 +6x + 4p = 0$ has product of roots equal to sum of roots ? (a) ½ (b) -1/2 (c) -3/2 (d) 5/2	(1)
5.	If 1 is the common root of the equation $ay^2 + ay + 3 = 0$ and $y^2 + y +b = 0$, then ab is equal to --------- (a) 5 (b) 3 (c) 4 (d) 9	(1)
6.	The roots of the equation $(c^2 – ab) x^2 -2(a^2 –bc) x+b^2 – ac = 0$ are equal , then prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$ Solve : $2(\frac{x}{x+1})^2 - 5(\frac{x}{x+1}) + 2 = 0$, $x\neq 1$	(4)
7.	Solve : $2(\frac{x}{x+1})^2 - 5(\frac{x}{x+1}) + 2 = 0$, $x\neq 1$	(4)
8.	An aeroplane left 30 min. late than its scheduled time and in order to reach its destination 1500 km. away in time , the pilot had to increase plane's speed y 250 km / hr. find the usual speed of the plane. Which value of pilot is depicted here ?	(4)
9.	The hypotenuse of a right triangle is $3\sqrt{10}$ cm. if smaller leg is tripled and the longer leg is doubled , new hypotenuse will be is $9\sqrt{5}$ cm. How long are the legs of the triangle ?	(4)

10.	Two pipes together can fill a tank in $11\frac{1}{9}$ minutes. If one of the pipes takes 5 min. more than the other to fill the tank separately , find the time in which each pipe would fill the tank separately.	(4)

ANSWER KEY:

1.	Answer key Given quadratic equation is $4x^2 - 2x + (c - 4) = 0$ On comparing with $ax^2 + bx + c$ a= 4 , b= 2 , c= c-4 Product of zeroes($\alpha.1/\alpha$)= c/a $\alpha.1/\alpha = (c - 4) /4$ $1 = (c - 4) /4$ $4 = c -4$ $4+4 = c$ $c = 8$ Hence, the value of c = 8	
2.	Given eq. is $x^2 - 2(p-2)x + (4p - 3 = 0$ Comparing with $ax^2 + bx + c = 0$ $a = 1$, $b = -2(p - 2)$ $c = 4p - 3$ for equal roots , D = 0 i.e. $b^2 - 4ac = 0$ or $(-2 (p - 2))^2 - 4.1.(p - 3) = 0$ or $p^2 8p +7 = 0$ $p = 7$ or $p = 1$	
3.	If S is the sum of roots and P is the product of roots then quadratic equation can be written as $x^2 - Sx + P = 0$ $\therefore$ $x^2 - (\sqrt{2} + 1) x + \sqrt{2} = 0$	
4.	$px^2 + 6x + 4p = 0$ comparing with $ax^2 + bx + c = 0$	

	sum of roots = $-\frac{b}{a} = -\frac{6}{p}$ product of roots = $\frac{c}{a} = \frac{4p}{p} = 4$ ATQ $-\frac{6}{p} = 4$ Or $p = -\frac{3}{2}$	
5.	1 is the common root of $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$ $\therefore$ $a(1)^2 + a(1) + 3 = 0$ and $(1)^2 + 1 + b = 0$ Or $a = -3$ and $b = -2$ $ab = 6$	
6.	Given equation is $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ And is quadratic equation in x for $c^2 - ab \neq 0$ Comparing with $Ax^2 + Bx + C = 0$, we get $A = c^2 - ab$, $B = -2(a^2 - bc)$ $C = b^2 - ac$ Now $D = B^2 - 4AC$ $= (-2(a^2 - bc))^2 - 4(c^2 - ab)(b^2 - ac)$ Or $D = 4a(a^3 - 3abc + b^3 + c^3)$ Since the roots are equal $\therefore$ $D = 0$ Hence either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$	
7.	Let $y = \frac{x}{x+1}$ $\therefore$ given equation becomes $2y^2 - 5y + 2 = 0$ $D = b^2 - 4ac = 9$ By quadratic formula $y = 2, \frac{1}{2}$ $\therefore$ $\frac{x}{x+1} = 2$ or $\frac{x}{x+1} = \frac{1}{2}$ $x = -2$ or $x = 1$	
8.	let usual speed be x km / hr distance = speed x time $\therefore$ usual time = 1500/x hr New speed = (x + 250) km/h New time = $\frac{1500}{x+250}$ hr	

	ATQ usual time of flight – new time of flight = 30 min. = ½ hr $\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$ Or $x^2 + 250x - 750000 = 0$ $\therefore$ x= 750 or x = -1000 (rejecting –ve value) Therefore usual speed = 750 km. Punctuality of the pilot is depicted.	
9.	Let the length of smaller leg be x cm and the larger leg be y cm. By using Pythagoras theorem, $H^2 = B^2 + P^2$ $x^2 + y^2 = (3\sqrt{10})^2$ $x^2 + y^2 = 9 \times 10$ $x^2 + y^2 = 90$ $y^2 = 90 - x^2$. …………(1) it is given that if the smaller leg is tripled and the longer leg is doubled, new hypotenuse will be $9\sqrt{5}$ cm Smaller leg = 3x longer leg = 2y By using Pythagoras theorem, $H^2 = B^2 + P^2$ $(3x)^2 + (2y)^2 = (9\sqrt{5})^2$	

	$9x^2 + 4y^2 = 81 \times 5$ $9x^2 + 4y^2 = 405$ $9x^2 + 4(90 - x^2) = 405$ [From eq 1] $9x^2 + 360 - 4x^2 = 405$ $x^2 = 9$ $x = \pm 3$ Since, length of a side can't be negative, so $x \neq -3$ Therefore, $x = 3$ Length of smaller leg be 3 cm On putting $x = 3$ in eq 1, $y^2 = 90 - x^2$ $y^2 = 90 - 3^2$ $y^2 = 81$ $y = \pm 9$ Since, length of a side can't be negative, so $y \neq -9$ Therefore, $y = 9$ Length of larger leg be 9 cm Hence, the length of smaller leg be 3 cm and the larger leg be 9 cm.	
10.	Let faster pipe take x minutes and the slower pipe will take (x+5) minutes. Time taken by both pipe to fill the tank = 100/9 minutes	

Portion of tank filled by faster pipe in 1 minute = $1/x$

Portion of tank filled by faster pipe in 1 minute = $1/(x+5)$

Portion of tank filled by both pipe in one 1 minute = $1/(100/9) = 9/100$

ATQ

$1/x + 1/(x+5) = 9/100$

$\Rightarrow (x+5 + x)/\{x(x+5)\} = 9/100$

$\Rightarrow (2x+5) / (x^2+5x) = 9/100$

$\Rightarrow 200x + 500 = 9x^2 + 45x$

$\Rightarrow 9x^2 + 45x - 200x - 500 = 0$

$\Rightarrow 9x2 - 155x - 500 = 0$

$\Rightarrow 9x2 - 180x + 25x - 500 = 0$

$\Rightarrow 9x (x - 20) + 25 (x - 20) = 0$

$\Rightarrow (9x + 25) (x - 20) = 0$

$x = -25/9$ and $x = 20$. (rejecting –ve value)
Therefore, $x = 20$ minutes.

Time taken by faster pipe to fill the tank = 20 minutes.

Time taken by slower pipe to fill the tank = 20+5 = 25 minutes.

CHAPTER-5 : ARITHMETIC PROGRESSIONS

SECTION-A (1 MARK QUESTION)

1	The 11th term of the AP: –5, (-5)/2, 0, 5/2, ...is (A) –20 (B) 20 (C) –30 (D) 30
2	Two APs have the same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4th terms is (A) –1 (B) – 8 (C) 7 (D) –9
3	The 4th term from the end of the AP: –11, –8, 5, ..., 49 is (A) 37 (B) 40 (C) 43 (D) 58
4	The value of n If the numbers n – 2, 4n – 1 and 5n + 2 are in AP is (A) 1 (B) 2 (C) 3 (D) 4
5	The value of the middle most term (s) of the AP : –11, –7, –3,..., 49 (A) 15,18 (B) 17,21 (C) 16,18 (D) 17,20

SECTION – B (2 MARK QUESTIONS)

1	Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.
2	The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP.
3	Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20. Find the common difference and the number of terms.
4	The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.
5	Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.
6	Verify that following is an AP and Then Write its next three terms. a + b, (a + 1) + b, (a + 1) + (b + 1), ….

SECTION-C (3 MARK QUESTIONS)

1	Which term of the AP −2, −7, −12, ... Will be −77? Find the sum of this AP up to the term −77.
2	In an AP, if $S_n = n (4n + 1)$, then find the AP.
3	In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.
4	Find the sum of first 17 terms of an AP whose 4^{th} and 9^{th} terms are – 15 and – 30, respectively.
5	If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, then find the sum of first 10 terms.
6	Find the sum of all the 11 terms of an AP whose middle most term is 30.

7	Find the sum of last ten terms of the AP: 8, 10, 12,--, 126.
8	Find the sum of first seven numbers which are multiples of 2 as well as of 9.

SECTION-D (4 MARK QUESTIONS)

1	Kanika was given her pocket money on Jan 1st, 2008. She puts Rs. 1 on day 1, Rs. 2 on day 2, Rs. 3 on day 3 and continued doing so till the end of the month, from this money into her piggy bank she also spent Rs. 204 of her pocket money, and found that at the end of the month she still had Rs. 100 with her. How much was her pocket money for the month.
2	Yasmeen saves Rs. 32 during the first month, Rs. 36 in the second month and Rs. 40 in the third month. If she continues to save in this manner, in how many moths will she save Rs. 2000?
3	The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the some of the first ten terms of this AP is 235, find the sum of its first twenty terms
4	**Find the** Sum of those integers from 1 to 500 which are multiples of 2 or 5.
5	An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the AP.
6	The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last terms to the product of the two middle terms is 7 : 15. Find the numbers.
7.	Solve the equation : 1 + 4 + 7 + 10 +...+ x =287

SOLUTIONS OF SUBJECT ENRICHMENT ACTIVITIES CLASS 10 ARITHMATIC PROGRESSION

SECTION-A

1	B
2	C
3	C
4	A
5	B

SECTION -B

1	Here, $d_1 = 7 - a$ .... (i) $d_2 = b - 7$ ....(ii) $d_3 = 23 - b$ ....(iii) $d_4 = c - 23$ ....(iv) Since a, 7, b, 23 and c are in AP. Therefore, $d_1 = d_2 = d_3 = d_4$ Taking (ii) and (iii), we get $b - 7 = 23 - b$ $\Rightarrow$ $2b = 30$ $\Rightarrow$ $b = 15$ Taking (i) and (ii), we get

	$7 - a = b - 7$ $\Rightarrow$ $7 - a = 15 - 7$ [Using b = 15] $\Rightarrow$ $7 - a = 8$ $\Rightarrow$ $a = -1$ Taking (iii) and (iv), we get $23 - b = c - 23$ $\Rightarrow$ $23 - 15 = c - 2$ [Using b = 15 $\Rightarrow$ $8 = c - 23$ $\Rightarrow$ $8 + 23 = c$ $\Rightarrow$ $c = 31$ Hence, $a = -1$, $b = 15$, $c = 31$.
2	Let the three terms in AP be $a - d, a, a + d$. So, $a - d + a + a + d = 33$ or $a = 11$ Also, $(a - d)(a + d) = a + 29$ i.e., $a^2 - d^2 = a + 29$ i.e., $121 - d^2 = 11 + 29$ i.e., $d^2 = 81$ i.e., $d = \pm 9$ So there will be two APs and they are : 2, 11, 20, ... and 20, 11, 2, ...
3	Let the first term of AP be a and common difference d. Using, $a_n = a + (n - 1)d$, we have: $a_5 = a + (5 - 1)d = 19$ [Given, $a_5 = 19$] $\Rightarrow$ $a + 4d = 19$ ....(i) Also, $a_8 = a + (8 - 1)d = a + 7d$ And, $a_{13} = a + (13 - 1)d = a + 12d$ According to question: $a_{13} - a_8 = 20$ $\Rightarrow$ $(a + 12d) - (a + 7d) = 20$ $\Rightarrow$ $5d = 20$ $\Rightarrow$ $d = 4$ Putting d = 4 in equation (i), we get $a + 4(4) = 19$ $\Rightarrow$ $a + 16 = 19$ $\Rightarrow$ $a = 19 - 16 = 3$

	So, required AP is given as: a, a + d, a + 2d, a + 3d, ... = 3, 3 + 4, 3 + 2(4), 3 + 3(4), …= 3, 7, 11, 15, ...
4	Let the first term and common difference of AP be a and d, respectively. According to the question, $a_5 + a_7 = 52$ and $a_{10} = 46$ $\Longrightarrow$ $(a + 4d) + (a + 6d) = 52$ and $a + 9d = 46$ [Using, $a_n = a + (n - 1)d$] $\Longrightarrow$ $2a + 10d = 52$ Or $a + 5d = 26$ ….(i) And $a + 9d = 46$ ….(ii) Subtracting equation (i) from equation (ii), we get: $4d = 20$ $\Longrightarrow$ $d = 5$ Putting d = 5, in equation (i) we get: $a + 5 (5) = 26$ $\Longrightarrow$ $a = 1$ So, Required AP is given as: a, a + d, a + 2d, a + 3d, ... = 1, 1 + 5, 1 + 2 (5), 1 + 3(5), ... = 1, 6, 11, 16, ...
5	Let the first term, common difference and number of terms of an AP are a, d and n, respectively. Given that, first term, a = 12. According to question, $a_7 = a_{11} - 24$ $\Longrightarrow$ $a + 6d = (a + 10d) - 24$ [Using, $a_n = a + (n - 1)d$] $\Longrightarrow$ $4d = 24$ $\Longrightarrow$ $d = 6$ Hence, 20th term of the AP, $a_{20} = a + 19d = 12 + 19(6) = 126$
6	Here, $a_1 = a + b$, $a_2 = (a + 1) + b$, $a_3 = (a + 1) + (b + 1)$ Now, $d_1 = a_2 - a_1$ $= [(a + 1) + b] - (a + b)$ $= a + 1 + b - a - b = 1$ $d_2 = a_3 - a_2$ $= [(a + 1) + (b + 1)] - [(a + 1) + b]$

	$= a + 1 + b + 1 - a - 1 - b = 1$ As $d_1 = d_2$, therefore given list of numbers forms an AP. Using, $a_n = a + (n - 1)d$, next three term are calculated as: $a_4 = a_1 + 3d = a + b + 3(1) = (a+2) + (b+1)$ $a_5 = a_1 + 4d = a + b + 4(1) = (a+2) + (b+2)$ $a_6 = a_1 + 5d = a + b + 5(1) = (a+3) + (b+2)$

SECTION-C

1	Let −77 be the nth term of AP −2, −7, −12,…. Here, first term, $a = -2$ and Common difference, $d = -7 - (-2) = -7 + 2 = -5$ Using formula for nth term of an AP, we have $a_n = a + (n-1)d$ $\Rightarrow \quad -77 = -2 + (n-1)(-5)$ $\Rightarrow \quad -75 = -(n-1) \times 5$ $\Rightarrow \quad (n-1) = 15$ $\Rightarrow \quad n = 16.$ So, the −77 is the 16th term of the given AP. Now, the sum of n terms of an AP is given as: $S_n = \frac{n}{2}[2a + (n-1)d]$ So, Sum of 16 terms i.e., up to the term −77 isgiven as: $S_{16} = \frac{16}{2}[2 \times (-2) + (n-1)(-5)]$ $= 8[-4 + (16-1)(-5)]$ $= 8(-4 - 75)$ $= 8 \times -79 = -632$
2	Given, $S_n = n(4n + 1)$ ….(i) We know that, the nth term if an AP is given as: $a_n = S_n - S_{n-1}$ $\Longrightarrow \quad a_n = n(4n + 1) - (n - 1)[4(n - 1) + 1]$ [Using (i)] $\Longrightarrow \quad a_n = 4n^2 + n - (n - 1)(4n - 3)$ $a_n = 4n^2 + n - 4n^2 + 4n + 3n - 3$ $\Longrightarrow \quad a_n = 8n - 3$ Put $n = 1$, $a_1 = 8(1) - 3 = 5$ Put $n = 2$, $a_2 = 8(2) - 3 = 16 - 3 = 13$ Put $n = 3$, $a_3 = 8(3) - 3 = 24 - 3 = 21$

	Hence, the required AP is 5, 13, 21, ...
3	Given, $S_n = 3n^2 + 5n$ …(i) And $a_k = 164$ the nth term if an AP is given as: $a_n = S_n - S_{n-1}$ $a_n = 3n^2 + 5n - 3(n-1)^2 - 5(n-1)$ [Using (i)] $a_n = 3n^2 + 5n - 3(n^2 + 1 - 2n) - 5n + 5$ $a_n = 3n^2 + 5n - 3n^2 - 3 + 6n - 5n + 5$ $a_n = 6n + 2$ Putting n = k in above equation and using equation (ii) , we get: $a_k = 6k + 2 = 164$ $\Longrightarrow$ $6k = 162$ $\Longrightarrow$ $k = 27$
4	let the first term, common difference and the number of terms of AP be a, d and n, respectively. Using formula for nth term of an AP, we have: $a_n = a + (n-1)d$ $\Longrightarrow$ 4^{th} term of AP, $a_4 = a + 3d$ $\Longrightarrow$ $a + 3d = -15$ [Given]….(i) Also, 9^{th} term of AP, $a_9 = a + 8d$ $\Longrightarrow$ $a + 8d = -30$ [Given]….(ii) Subtracting equation (ii) from equation (iii), We get d=-3 put the value of d= -3 in eq we get a=-6 now sum of first 17 terms of an ap = -510
5	Let the first term and common difference of AP be a and d respectively. Given that, sum of first 6 terms of AP, $S_6 = 36$ And, sum of first 16 terms of AP, $S_{16} = 256$ As sum of first n terms of an AP is given as:

	$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ ....(i) Therefore, $S_6 = 36$ can be written as: $\Rightarrow \frac{6}{2}\left[2a+(6-1)d\right] = 36$ $\Rightarrow 2a+5d = 12$ ...(ii) Also, $S_{16} = 256$ can be written as: $\Rightarrow \frac{16}{2}\left[2a+(16-1)d\right] = 256$ $\Rightarrow 2a+15d = 32$ ...(iii) On subtracting equation (ii) from equation (iii), we get $10d = 20$ $\Rightarrow d = 2$ Putting $d = 2$ in equation (ii), we get: $2a + 5(2) = 12$ $\Rightarrow 2a = 12 - 10 = 2$ $\Rightarrow a = 1$ Thus, sum of first 10 terms is given as: $S_{10} = \frac{10}{2}\left[2a+(10-1)d\right]$ [Using (i)] $= 5\left[2(1)+9(2)\right] = 5(2+18)$ $= 5\times 20 = 100$
6	Since the total number of terms, n = 11, which is odd $\therefore$ Middle most term $= \frac{(n+1)}{2}$ th term $= \left(\frac{11+1}{2}\right) =$ 6th term Given that, $a_6 = 30$ Using formula for nth term of an AP, we have: $a_n = a + (n-1)d$ $\Rightarrow a_6 = a + (6-1)d$ $\Rightarrow a + 5d = 30$ ...(i) Now, sum of n terms of an AP is given as: $\therefore S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ $S_{11} = \frac{11}{2}[2a+10d]$ $= 11(a+5d)$ $= 11\times 30 = 330$ [Using(i)]
7	For finding, the sum of last ten terms, write the given AP in reverse order.

	So, the new AP becomes: 126, ..., 12, 10, 8 ....(i) Here, first term, a = 126 And, common difference, d = 10 – 10 = –2 As, sum of first n terms of an AP is given as: $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ Therefore, sum of first 10 terms of AP (i) is given as: $S_{10} = \frac{10}{2}\left[2a+(10-1)d\right]$ $= 5\left[2(126)+9(-2)\right]$ $= 5(252-18)$ $= 5\times 234 = 1170$ Hence, the sum of last ten terms of the AP 8, 10, 12, ..., 126 is 1170.
8	To find, the list of numbers which are multiples of 2 as well as of 9, we first take the LCM of 2 and 9 which is 18. So, the series of numbers which are multiples of 2 as well as of 9 is: 18, 36, 54,.... Here, first term, a = 18 And, common difference, d = 36 – 18 = 18 As, sum of first n terms of an AP is given as: $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ ∴ Sum of first seven numbers of AP 18, 36, 54,.... is given as: $S_7 = \frac{7}{2}\left[2(18)+(7-1)18\right]$ $= \frac{7}{2}[36+6\times 18] = 7(18+3\times 18)$ $= 7\times 72 = 504$

SECTION-D

1	Let her pocket money be Rs. x. Out of Rs. x, money put in piggy bank from Jan. 1 to Jan. 31 = 1 + 2 + 3 + 4 + … + 31. Which forms an AP in first term, common difference and number of terms being 1, 1 and 31 respectively.

	$\therefore$ Sum of first 31 terms is given as: $S_{31} = \frac{31}{2}[2\times1+(31-1)\times1]$ $\left[\text{Using, } S_n = \frac{n}{2}[2a+(n-1)d]\right]$ $= \frac{31}{2}(2+30) = \frac{31\times32}{2}$ $= 31\times16 = 496$ Therefore total money deposited in piggy bank = Rs. 496 Also, money spent by Kanika = Rs. 204 And money left at the end of the month = Rs. 100 $\therefore \quad x - 496 - 204 = 100$ $\Rightarrow \quad x - 700 = 100$ $\Rightarrow \quad x = \text{Rs. } 800$ Hence, Rs. 800 was her pocket money for the month.
2	Amount saved during the first month = Rs. 32 Amount saved during the second month = Rs. 36 Amount saved during the third month = Rs. 40 Thus we have an arithmetic progression 32, 36, 40, ... Here, first term, a = 32, And, common difference, d = 36 – 32 = 4 Let Rs. 2000 will be saved during n months. i.e., S_n = Rs. 2000 We know that, sum of first n terms of an AP is given as: $S_n = \frac{n}{2}[2a+(n-1)d]$ $\Rightarrow \quad 2000 = \frac{n}{2}[2\times32+(n-1)\times4]$ $\Rightarrow \quad 2000 = \frac{n}{2}(64+4n-4)$ $\Rightarrow \quad 2000 = \frac{n}{2}(60+4n)$ $\Rightarrow \quad 2000 = \frac{4n}{2}(15+n)$ $\Rightarrow \quad 1000 = n(15+n)$ $\Rightarrow \quad 1000 = 15n+n^2$ $\Rightarrow \quad n^2+15n-1000 = 0$ Solving above equation by factorsation method, we get: $n^2+40n+25n-1000 = 0$ $\Rightarrow \quad n(n+40)-25(n+40) = 0$ $\Rightarrow \quad (n+40)(n-25) = 0$ $\Rightarrow \quad (n+40) = 0 \text{ or } (n-25) = 0$ $\Rightarrow \quad n = 25 \text{ or } -40$ Ignore n = −40 as a month cannot be negative. Hence, Rs. 2000 will be saved in 25 months.
3	Let the first term, common difference and the number of terms of given AP be a, d and n, respectively.

	As sum of n term of an AP is given as: $$S_n = \frac{n}{2}\left[2a+(n-1)d\right] \quad ....(i)$$ $\therefore$ Sum of first five terms of an AP is given as, $$S_5 = \frac{5}{2}\left[2a+(5-1)d\right]$$ $$\Rightarrow \quad S_5 = \frac{5}{2}(2a+4d) = 5(a+2d)$$ $$\Rightarrow \quad S_5 = 5a+10d \quad ....(ii)$$ And sum of first seven terms of an AP is given as, $$S_7 = \frac{7}{2}\left[2a+(7-1)d\right] \quad [\text{Using(i)}]$$ $$\Rightarrow \quad S_7 = \frac{7}{2}\left[2a+6d\right] = 7(a+3d)$$ $$\Rightarrow \quad S_7 = 7a+21d \quad ...(iii)$$ According to question: $$S_5+S_7 = 167$$ $$\Rightarrow \quad 5a+10d+7a+21d = 167$$ $$\Rightarrow \quad 12a+31d = 167 \quad ....(iv)$$ Also given that, sum of first ten terms of this AP, $S_{10} = 235$ $$\Rightarrow \quad \frac{10}{2}\left[2a+(10-1)d\right] = 235$$ $$\Rightarrow \quad 5(2a+9d) = 235$$ $$\Rightarrow \quad 2a+9d = 47 \quad ....(v)$$ Applying $6\times(v)-(iv)$, we get $$\begin{array}{r} 12a+54d = 282 \\ 12a+31d = 167 \\ - \quad - \quad - \\ \hline 23d = 115 \end{array}$$ $$\Rightarrow \quad d = 5$$ Putting value of d in equation (v), we get $$2a+9(5) = 47$$ $$\Rightarrow \quad 2a+45 = 47$$ $$\Rightarrow \quad 2a = 2$$ $$\Rightarrow \quad a = 1$$ Thus, sum of first twenty terms of this AP is given as: $$S_{20} = \frac{20}{2}\left[2a+(20-1)d\right]$$ $$S_{20} = 10\left[2\times(1)+19\times(5)\right] = 10(2+95)$$ $$S_{20} = 10\times 97 = 970$$ Hence, the required sum of first twenty terms of AP is 970.
4	Since, multiples of 2 or 5 from 1 to 500 = Multiple of 2 from 1 to 500 + Multiples of 5 from 1 to 500 – Multiple of 10 from 1 to 500 = (2, 4, 6, ..., 500) + (5, 10, 15, ..., 500) – (10, 20, ..., 500) .....(i) Let AP_1 = 2, 4, 6, ..., 500 AP_2 = 5, 10, 15, ..., 500 And AP_3 = 10, 20, ..., 500

Let number of terms in AP_1, AP_2 and AP_3 be n_1, n_2 and n_3 respectively.
Now formula for last term of an AP is given as:

l= an + (n – 1)d ....(ii)

For AP_1,

$500 = 2 + (n_1 - 1)2$ ....[Using (ii)]

$\Rightarrow$ $250 = 1 + (n_1 - 1)$

$\Rightarrow$ $n_1 = 250$

For AP_2,

$500 = 5 + (n_2 - 1)5$ ....[Using (ii)]

$\Rightarrow$ $100 = 1 + (n_2 - 1)$

$\Rightarrow$ $n_2 = 100$

For AP_3,

$500 = 10 + (n_3 - 1)10$ ....[Using (ii)]

$\Rightarrow$ $250 = 1 + (n_3 - 1)$

$\Rightarrow$ $n_3 = 50$

Thus, from equation (i),

Sum of Multiples of 2 or 5 from 1 to 500

= Sum of (2, 4, 6, ... 500) + Sum of (5, 10, ... 500) – Sum of (10, 20, ... 500)

$$=\frac{n_1}{2}[2+500]+\frac{n_2}{2}[5+500]-\frac{n_3}{2}[10+500] \qquad \left[\because S_n=\frac{n}{2}(a+l)\right]$$

$$=\frac{250}{2}\times 502+\frac{100}{2}\times 505-\frac{50}{2}\times 510$$

$$=125\times 502+505\times 50-25\times 510$$

$$=62750+25250-12750$$

$$=88000-12750=75250$$

5 Given, total number of terms, n = 37, which is odd.

$$\therefore \text{Middle term} = \left(\frac{37+1}{2}\right)\text{th term} = 19^{\text{th}}\text{ term}$$

So, the three middle most terms are 18^{th}, 19^{th} and 20^{th} terms.

By given condition,

Sum of the three middle most terms = 225

i.e., $a_{18}+a_{19}+a_{20}=225$

$\Rightarrow$ $(a+17d)+(a+18d)+(a+19d)=225$ $\quad [\because a_n = a+(n-1)d]$

$\Rightarrow$ $3a+54d=225$

$\Rightarrow$ $a+18d=75$ ....(i)

Also, given that, sum of last three terms = 429

$\Rightarrow$ $a_{35}+a_{36}+a_{37}=429$

$\Rightarrow$ $(a+34d)+(a+35d)+(a+36d)=429$

$\Rightarrow$ $3a+105d=429$

$\Rightarrow$ $a+35d=143$ ....(ii)

On subtracting equation (i) from equation (ii), we get

$17d=68$

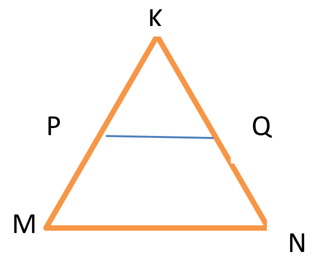
$\Rightarrow$ $d=4$

	Putting value of d in equation (i), we get: $a+18(4)=75$ $\Rightarrow \quad a = 75-72$ $\Rightarrow \quad a = 3$ $\therefore$ Required AP is a, a + d, a + 2, a + 3d,.... i.e., 3, 3 + 4, 3 + 2(4), 3 + 3(4),...., i.e., 3, 7, 3 + 8, 3 + 12,...., i.e., 3, 7, 11, 15, ...
6.	Let the four consecutive numbers in AP be a –3d, a – d, a + d, a + 3d. So, a –3d + a – d + a + d + a + 3d = 32 or 4a = 32 or a = 8 $=\frac{(a+3d)(a-3d)}{(a-d)(a+d)} = \frac{7}{15}$ $=\frac{a^2-9d^2}{a^2-d^2}$ $15a^2-135d^2=7a^2-7d^2$ $8a^2-128d^2=0$ $d^2=4$ $d=\pm 2$ so the ap is 2,6,10,14
7.	Here, 1, 4, 7, 10, ..., x form an AP with a = 1, d = 3 $a_n = x$ We have, $a_n = a + (n-1)d$ So, $x = 1 + (n-1) \times 3 = 3n-2$ Also, $S_n = \frac{n}{2}(a+l)$ So, $287 = \frac{n}{2}(1+x)$ $=\frac{n}{2}(1+3n-2)$ or, $574 = n(3n-1)$ or, $3n^2 - n - 574 = 0$ Therefore, $n = \frac{1+\sqrt{1+6888}}{6}$ $n=\frac{1\pm 83}{6}$ $n=14$ or $\frac{-41}{3}$=rejected n=14 Therefore, $x = 3n-2 = 3 \times 14 - 2 = 40$.

CHAPTER 6- TRIANGLES

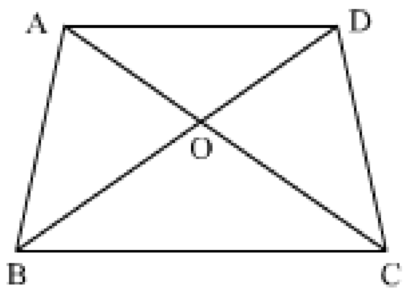
(2 Marks)

1. In figure PQ||MN and $\frac{KP}{PM} = \frac{4}{13}$, KN = 20.4cm. Find KQ.



2. In triangle ABC , DE||BC. If AD = x, DB = x-2, AE = x+2 and EC = x – 1, find the value of ‘x’.
3. AD is the bisector of $\angle A$, if BD = 4 cm, DC = 3cm and AB = 6cm determine AC.

4. ABCD is a quadrilateral and diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{OB}{OD}$.Show that ABCD is a trapezium.



(3 – MARKS)

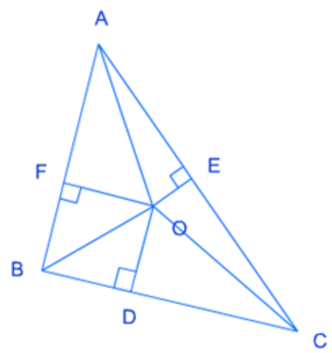
1 ABC is a right-angled triangle, right angled at B. AD and CE are two medians drawn from A and C respectively. If AC = 5 cm, and AD = (3 √ 5)/ 2 cm, find the length of CE.

2.In the given figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC

and OF ⊥ AB. Show that

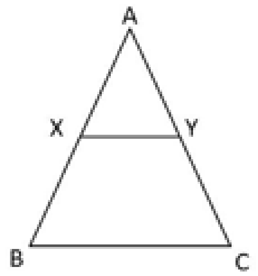
(a) $OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2 = AF^2 + BD^2 + CE^2$

(b) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



3.If ABC and DBC are two triangles on the same base BC and AD intersects BC at O, show that $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$

4. In ΔABC, XY || AC and area of ΔBXY = area of quadrilateral XYCA. Find AX/XB



⇒ar (ΔABC) = 2.ar (ΔBXY) ----------------(1)

XY || AC and BA is a transversal.

⇒∠BXY = ∠BAC --------------------------- (2)

So, In ΔBAC and ΔBXY,

∠XBY = ∠ABC (common angle)

∠BXY = ∠BAC [from equation (2)]

⇒ ΔBAC ~ ΔBXY

⇒ar(ΔBAC) / ar(ΔBXY) = BA^2 / BX^2

⇒ BA = √2 BX

⇒ BA = √2 (BA – AX)

⇒ (√2 – 1) BA = √2 AX

⇒ AX/XB = (√2 – 1) / √2

5. Given: BL and CM are medians of the ΔABC in which ∠A = 90° .

To Prove: $4 (BL^2+ CM^2) = 5 BC^2$

(4 MARKS)

1. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

2. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

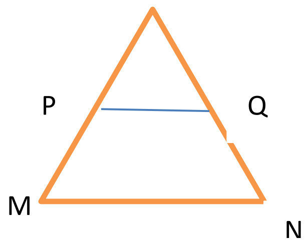
3. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after 1.5 hours?

ANSWERS

(2 MARKS)

1 Solution: In ΔKMN,

$$\frac{KP}{MP} = \frac{KQ}{QN} \quad (\because PQ \| MN \text{ given})$$



$$\frac{KP}{MP} = \frac{KQ}{KN - KQ}$$

$$\frac{4}{13} = \frac{KQ}{20.4 - KQ}$$

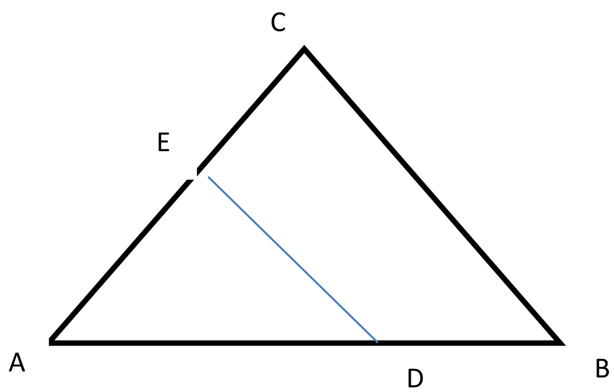
$\Rightarrow 4(20.4 – KQ) = 13 KQ$

$\Rightarrow 81.6 – 4KQ = 13 KQ$

$\Rightarrow KQ = 4.8cm$

2 Solution: In ΔABC,

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{(byThales' theorem)}$$



$$\frac{x}{x-2} = \frac{x+2}{x-1}$$

$x(x-1) = (x-2)(x+2)$

$x^2 – x = x^2 – 4$

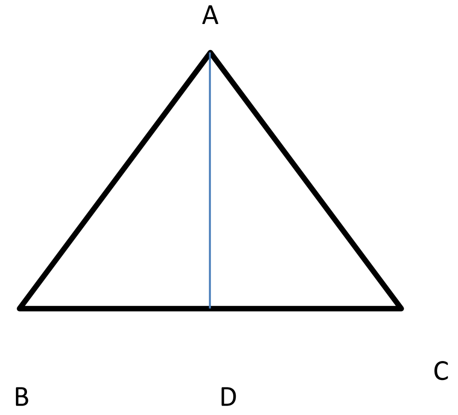
$x = 4$

3Solution: In ΔABC , AD is the bisector of $\angle A$

$$\therefore \frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{4}{3} = \frac{6}{AC}$$

$$\Rightarrow AC = \frac{3 X 6}{4} = 4.5 \text{ cm}$$



4Sol: Given that ABCD is a quadrilateral and diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{OB}{OD}$

IN Δ AOD and ΔBOC $\frac{AO}{OC} = \frac{BO}{OD}$

$\angle$ AOD = $\angle$COB

Thus Δ AOC ~ ΔBOC (SAS similarity criterion)

$\Rightarrow$ $\angle$OAD = $\angle$ OCB ……………..1

Now transversal AC intersect AD and BC such the

$\angle$CAD = $\angle$ACB (from …..1) (alternate opposite angles)

So AD || BC

Hence ABCD is a trapezium

(3 MARKS)

1Solution: In right angled triangle

Let BD = DC = x

AE = BE = y

In right angled triangle ABC

$AB^2 + BC^2 = AC^2$

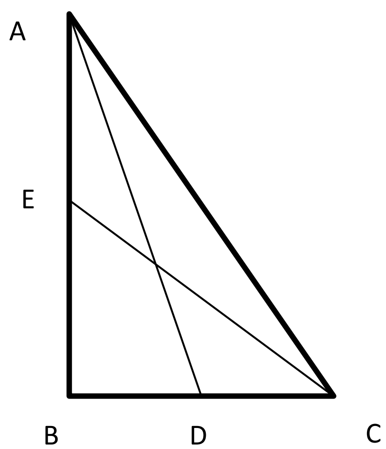
$(2y)^2 + (2x)^2 = 25$

$4y^2 + 4x^2 = 25$ ….(i)

In right angled triangle ABD

$AB^2 + BD^2 = AD^2$

$$4y^2 + x^2 = \left(\frac{3\sqrt{5}}{2}\right)^2$$



$4y^2 + x^2 = \frac{45}{4}$

$16y^2 + 4x^2 = 45$ ……..(ii)

Subracting equation (i) from (ii)

$12y^2 = 20$

$y^2 = \frac{5}{3}$

Put this value in equation (i)

$4 \times \frac{5}{3} + 4x^2 = 25$

$4x^2 = 25 - \frac{20}{3}$

$x^2 = \frac{55}{12}$

Now in right triangle BEC

$CE^2 = BE^2 + BC^2$

$= y^2 + (2x)^2$

$= y^2 + 4x^2$

$= \frac{5}{3} + 4 \times \frac{55}{12}$

$= 20$

$CE = 2\sqrt{5}$ cm

2(a)Solution: In Δ AFO; $AF^2 = OA^2 - OF^2$

In Δ BDO; $BD^2 = OB^2 - OD^2$

In Δ CEO; $CE^2 = OC^2 - OE^2$

Adding the above three equations, we get;

$AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$ proved

(b) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Solution: In Δ AEO; $AE^2 = OA^2 - OE^2$

In Δ CDO; $CD^2 = OC^2 - OD^2$

In Δ BFO: $BF^2 = OB^2 - OF^2$

Adding the above three equations, we get;

$AE^2 + CD^2 + BF^2 = OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2$

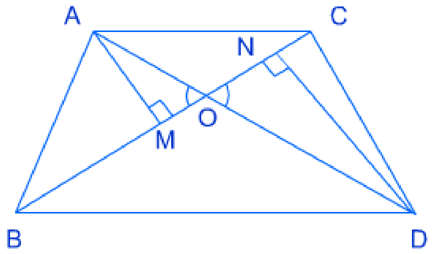
From the previous solution, we also have;

$AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2$

Comparing the RHS of the above two equations, we get;

$$AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$$

3Solution: Let us draw altitudes AM and DN on BC; respectively from A and D



$$\frac{ar(ABC)}{ar(DBC)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$$

$$= \frac{AM}{DN}$$

In ΔAMO and ΔDNO;

∠ AMO = ∠ DNO (Right angle)

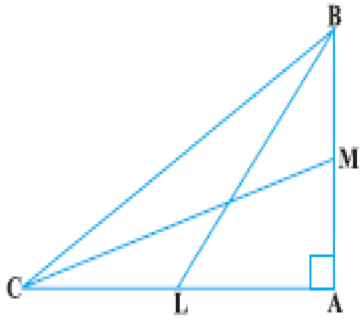
∠ AOM = ∠ DON (Opposite angles)

Hence; ΔAMO ~ ΔDNO

$$\frac{AM}{DN} = \frac{AO}{DO}$$

Hence; $$Or, \frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$$

5 Proof:



From ΔABC,

$BC^2 = AB^2 + AC^2$ (Pythagoras Theorem) (1)

From ΔABL,

$BL^2 = AL^2 + AB^2$

$BL^2 = \frac{AC2}{4} + AB^2$ (L is the mid-point of AC)

or, $4BL^2 = AC^2 + 4AB^2$………………………..(2)

From Δ□CMA,

$CM^2 = AC^2 + AM^2$

or, $CM^2 = AC^2 + \frac{AB2}{4}$ (M is the mid-point of AB)

or $4\ CM^2 = 4\ AC^2 + AB^2$…………………(3)

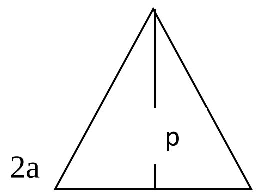
Adding (2) and (3),

$4\ (BL^2 + CM^2) = 5\ (AC^2 + AB^2)$

i.e., $4\ (BL^2 + CM^2) = 5\ BC^2$[From (1)]

(4 MARKS)

1 Solution: In case of an equilateral triangle, an altitude will divide the triangle into two congruent right triangles. In the right triangle thus formed, we have;



Hypotenuse = One of the sides of the equilateral triangle = 2a

Perpendicular = altitude of the equilateral triangle = p

Base = half of the side of the equilateral triangle = a

Using Pythagoras theorem, the perpendicular can be calculated as follows:

$p^2 = h^2 – b^2$

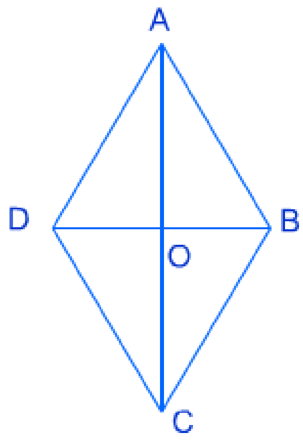
Or, $p^2 = (2a)^2 – a^2$

Or, $p^2 = 4a^2 – a^2 = 3a^2$

Or, $p = a\sqrt{3}$

2 Solution:

Given: ABCD is a rhombus in which diagonals AC and BD intersect at point O.



To Prove: $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

Proof: In Δ AOB; $AB^2 = AO^2 + BO^2$

In Δ BOC; $BC^2 = CO^2 + BO^2$

In Δ COD; $CD^2 = CO^2 + DO^2$

In Δ AOD; $AD^2 = DO^2 + AO^2$

Adding the above four equations, we get;

$AB^2 + BC^2 + CD^2 + AD^2$

$= AO^2 + BO^2 + CO^2 + BO^2 + CO^2 + DO^2 + DO^2 + AO^2$

Or, $AB^2 + BC^2 + CD^2 + AD^2 = 2(AO^2 + BO^2 + CO^2 + DO^2)$

Or, $AB^2 + BC^2 + CD^2 + AD^2 = 2(2AO^2 + 2BO^2)$

(Because AO = CO and BO = DO)

Or, $AB^2 + BC^2 + CD^2 + AD^2 = 4(AO^2 + BO^2)$ ………(1)

Now, let us take the sum of squares of diagonals;

$AC^2 + BD^2 = (AO + CO)^2 + (BO + DO)^2$

$= (2AO)^2 + (2BO)^2$

$= 4AO^2 + 4BO^2$ ……(2)

From equations (1) and (2), it is clear;

$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

Proved

3Solution: Distance covered by the first plane in 1.5 hours = 1500 km

Distance covered by the second plane in 1.5 hours = 1800 km

The position of the two planes after 1.5 hour journey can be shown by a right triangle and we need to find the hypotenuse to know the aerial distance between them.

Here; h = ? p = 1800 km and b = 1500 km

From Pythagoras theorem;

$h^2 = p^2 + b^2$

Or, $h^2 = 1800^2 + 1500^2$

$= 3240000 + 2250000 = 5490000$

Or, $h = 300\sqrt{61}$ km

CHAPTER-7: COORDINATE GEOMETRY

- **LINES (In two-dimensions)**
- **Concepts of coordinate geometry, graphs of linear equations.**
- **Distance formula.**
- **Section formula (internal division).**
- **Area of a triangle.**

IMPORTANT POINTS TO REMEMBER

❖ **Distance formula**
Distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by
$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

❖ **The distance of a point P(x, y) from the origin O(0, 0) is given by OP = $\sqrt{x^2 + y^2}$**

❖ **Section formula**
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points and P(x,y) divides AB internally in the ratio $m_1 : m_2$, then
$$x = \frac{m_1x_2 + m_2x_1}{m_1+m_2}, y = \frac{m_1y_2 + m_2y_1}{m_1+m_2}$$

❖ **The coordinates of the mid-point of the line segment joining the points P (x_1, y_1) and Q (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$**

❖ **Area of triangle**
Area of ΔABC whose vertices are $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ is
$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

❖ **Main mistake that students make is in calculation.**

IMPORTANT QUESTIONS FROM CHAPTER

1. If three points (0,0), (3,$\sqrt{3}$) and (3, k) form an equilateral triangle find the value of k

   (A) 2, (B) -3, (C) -$\sqrt{3}$, (D) - $\sqrt{2}$.
2. The triangle whose vertices are (0, 0), (2.7, 0) & (0, 4.9) is a/an

   (A) equilateral triangle, (B) Right angle triangle, (c) isosceles triangle, (D) obtuse angled triangle.
3. If the points (-2, -5), (2, -2) & (8, p) are collinear then the value of p is…………

4. The distance of the point (h, k) from x-axis is

   (A) h units, (B) $|h|$units, (C) k units, (D) $|k|$units.
5. The distance of the point (α, β) from y- axis is

   (A) α, (B) $|\alpha|$, (C) β, (D) $|\beta|$.
6. Line formed by joining (-1, 1) and (5, 7) is divided by a line x + y = 4 in the ratio of

   (A)1: 2, (B)1: 3 ,(C)3: 4, (D)1: 4
7. Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.
8. If P (2, 1), Q (4, 2), R (5, 4) and S (3, 3) are vertices of quadrilateral, find the area of quadrilateral PQRS.
9. Find the linear relation between x and y such that p (x, y) is equidistant form the points A (1, 4) and B (-1, 2).
10. Find the points on y-axis which is equidistant from the points (5, -2) and (-3, 2).
11. Write the coordinates of a point P on x-axis which is equidistant from the points A(-2, 0) and B (6, 0).
12. Name the type of triangle formed by A (2, 7), B (4, 11) and C (6, 15).
13. Find the ratio in which P (4, m) divides the line segment joining the points A (2, 3) and B (6, -3). Hence find m.
14. If the point C (-1, 2) divides the line segment AB in the ratio 3: 4 where the co-ordinates of A are (2, 5). Find the co-ordinates of B.
15. If A (-2, 1) B (a, 0) C (4, b) and D (1, 2) are the vertices of a parallelogram ABCD. Find the value of a and b.
16. If the two vertices of a parallelogram are A (3, 2) and B (1, 0) and the diagonals cut at (2, -5) find the other vertices of the parallelogram.
17. If the area of the triangle with vertices (x, 0), (1, 1) and (0, 2) is 4 square units, then find the value of x.
18. Prove that (-2, 3), (8, 3) and (6, 7) are the vertices of right angled triangle.
19. If a is the length of one side of an equilateral triangle ABC, base BC lies on x-axis and vertex B is at the origin, find the coordinates of the vertices of the triangle ABC.
20. Find the area of the triangle formed by the line 5x -3y +15 = 0 with co-ordinates axes
21. If points (-2, 1), (a, b) and (4, -1) are collinear and a – b = 1,then find the value of a and b.
22. A(0, 3), B(-1, -2) and C(4, 2) are vertices of a ΔABC. D is a point on side BC such that $\frac{BD}{DC} = \frac{1}{2}$. P is a point on AD such that AP = $\frac{2\sqrt{3}}{3}$ units. P is a point on AD such that AP = $\frac{2\sqrt{5}}{3}$ Find the coordinates of P.
23. A(6, 1), B(8, 2) and C (9, 4) are three vertices of a parallelogram ABCD. If E is a midpoint of DC, find the area of the ΔADE.

24. The points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of ΔABC.
    a. The median from A meets BC at D. Find the coordinates of the points D.
    b. Find the coordinates of the points P on AD such that AP: PD= 2:1.
    c. Find the coordinates of points Q and R on medians BE and CF, respectively such that BQ: QE = 2:1 and CR: RF=2:1.
    d. What are coordinates of the centroid of the triangle ABC?
25. The midpoint P of the line segment joining the points A(-10, 4) and B(-2, 0) lies on the line segment joining the points C(-9, -4) and D(-4, y). find the ratio in which P divides CD. Also find the value of y.
26. The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, -3). The origin is the midpoint of the base. Find the coordinates of the points A and B. Also find the coordinates of another point D such that BACD is a rhombus.
27. Prove that the area of the triangle with vertices (t, t-2), (t+2, t+2) and (t+3, t) is independent of t.
28. The co-ordinates of the points A, B and C are (6, 3), (-3, 5) and (4, -2) respectively. P(x, y) is any point in the plane. Show that $\frac{\text{ar}(\Delta PBC)}{\text{ar}(\Delta ABC)} = \frac{x+y-2}{7}$

ANSWERS

1. C
2. B
3. 5/2
4. D
5. B
6. A
7. X+3Y-7=0
8. Area of $\Delta PQR = \frac{1}{2}|2(2-4)+4(4-1)+5(1-2)|$

   $=\frac{1}{2}|2\times -2+4\times 3+5\times -1| = \frac{1}{2}|-4+12-5| = \frac{3}{2}\text{sq. unit.}$

   Area of $\Delta PRS = \frac{1}{2}|2(4-3)+5(3-1)+3(1-4)| = \frac{1}{2}|2+10-9| =$

   $\frac{3}{2}\text{sq. units.}$

   ∴ Area of quadrilateral = Area of ΔPQR + Area of ΔPRS $= \frac{3}{2}+\frac{3}{2} = 3\text{ sq. units.}$
9. P(x,y) is equidistant from the points A(1,4) and B(-1,2)

   PA = PB

   $\sqrt{(x-1)^2+(y-4)^2} = \sqrt{(x+1)^2+(y-2)^2}$

   Squaring both sides, we get

$(x-1)^2+(y-4)^2=(x+1)^2+(y-2)^2$

$x^2-2x+1+y^2-8y+16=x^2+2x+1+y^2-4y+4$

$-2x+17-8y=2x-4y+5$

$\Rightarrow\ -4\mathrm{x}-4\mathrm{y}=-12$

$\Rightarrow\ \mathrm{x}+\mathrm{y}=3$

10. Let the point on y-axis be (0,a)

Now distance of this point from (5, -2) is equal to distance from point (-3, 2)

i.e., $\sqrt{5^2+(-2-a)^2}=\sqrt{3^2+(a-2)^2}$

Squaring and simplifying, we get

$25+4+a^2+4a=9+a^2+4-4a$

$8a=-16$

$a=-2$

So point is (0, -2)

11. Let coordinates of the point P are (x, 0)

ACOORDING TO QUESTION

$\mathrm{AP}=\mathrm{BP}$

$\sqrt{(x+2)^2+(0-0)^2}=\sqrt{(x-6)^2+(0-0)^2}$

$(x+2)^2=(x-6)^2$

$x^2+4+4x=x^2+36-12x$

$16x=32\ \Rightarrow x=2$

∴ Coordinates of the point P are (2,0)

12. $\mathrm{AB}=\sqrt{(4-2)^2+(11-7)^2}$

$=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$ units

$\mathrm{BC}=\sqrt{(6-4)^2+(15-11)^2}$

$=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$ units

$\mathrm{AC}=\sqrt{(6-2)^2+(15-7)^2}$

$=\sqrt{16+64}=\sqrt{80}=4\sqrt{5}$ units

∴ AB + BC = AC

∴ AB, BC and AC cannot be the sides of any triangle.

(∴ sum of two sides of a triangle is more than the third side)

A, B and C are collinear

13. Let, P(4, m) divide A(2, 3) and B(6, -3) in the ratio $\mathrm{m_1}:\mathrm{m_2}$,

[By using section formula, $\mathrm{x}=\frac{\mathrm{m_1x_2+m_2x_1}}{\mathrm{m_1+m_2}},\mathrm{y}=\frac{\mathrm{m_1y_2+m_2y_1}}{\mathrm{m_1+m_2}}$]

$$4=\frac{\mathrm{m_1}\times 6+\mathrm{m_2}\times 2}{\mathrm{m_1}+\mathrm{m_2}}$$

$OR\ \ 4\mathrm{m_1}+4\mathrm{m_2}=6\mathrm{m_1}+2\mathrm{m_2}$ OR $4\mathrm{m_2}-2\mathrm{m_2}=6\mathrm{m_1}-4\mathrm{m_1}$

$2\mathrm{m_2}=2\mathrm{m_1}\ \ \therefore \mathrm{m_1}:\mathrm{m_2}=1:1$

Also, $\mathrm{m}=\frac{\mathrm{m_1}\times(-3)+\mathrm{m_2}\times 3}{\mathrm{m_1+m_2}}==\frac{1\times(-3)+1\times 3}{2}=\frac{-3+3}{2}=\frac{0}{2}=0\Rightarrow\ \mathrm{m}=0$

14. $\frac{3\times x+4\times 2}{3+4}=-1$

$\frac{3x+8}{7}=-1$ OR $3x+8=-7 \Rightarrow 3x=-15 \Rightarrow x=-5$

$\frac{3\times y+4\times 5}{3+4}=2 \Rightarrow \frac{3y+20}{7}=2 \Rightarrow 3y+20=14 \Rightarrow 3y=-6 \Rightarrow y=-2$

∴ Coordinates of B are (-5, -2).

15. Mid-point for diagonal AC = $(1, \frac{1+b}{2})$

Mid-point for diagonal BD = $(\frac{1+a}{2}, 1)$

So, $1=\frac{1+a}{2}$	Also, $\frac{1+b}{2}=1$
$2=1+a$	$1+b=2$
$a=1$	$b=1$
∴ a = 1, b = 1	

$AB=BC=CD=AD=\sqrt{10}$ units

16. Let ABCD be a parallelogram, diagonals AC and BD intersect at O.

Let A(3, 2), B(1, 0) and O(2, -5) are coordinates
Let coordinates of C are (a, b) and coordinates of D are (x, y). As diagonals of parallelogram bisect each other at O. So, O is the mid-point of AC and BD.

$2=\frac{3+a}{2}$ and $-5=\frac{2+b}{2}$

a = 1 and b = -12

Also, $\frac{1+x}{2}=2$ and $\frac{0+y}{2}=-5$

x = 3 and y = - 10

∴ Coordinates are (1, -12) and (3, -10)

17. Area of a triangle = $\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$\Rightarrow \frac{1}{2}|x(1-2)+1(2-0)+0(0-1)|=4$

$\Rightarrow |-x+2|=8$

$\Rightarrow -x+2=\pm 8$

$\Rightarrow -x=8-2 \text{ or} -x=-8-2$

$\Rightarrow x=-6 \text{ or } x=10$

18. Let A(-2, 3), B(8, 3) and C(6, 7) are the vertices of triangle

Then AB = 10 units

BC = $\sqrt{20}$

AC = $\sqrt{80}$

As $AC^2 + BC^2 = AB^2$

So triangle ABC is right angled triangle.

19. Equilateral triangle of side a units

Suppose B is at origin, therefore the coordinates of B are (0, 0)
And C lies on x-axis, so coordinates of C are (a, 0)
As, AB = BC = AC

$$\sqrt{(x-0)^2+(y-0)^2}=a=\sqrt{(x-a)^2+(y-0)^2}$$

Squaring, we get

$x^2+y^2=a^2=x^2+a^2-2ax+y^2$ ………… (i)

$\Rightarrow 2ax-a^2=0 \Rightarrow a(2x-a)=0$

$\Rightarrow \mathrm{a} = 0$ or $2\mathrm{x} - \mathrm{a} = 0$, but $\mathrm{a} \neq 0$

$\therefore x = \dfrac{a}{2}$

Substituting $x = \frac{a}{2}$ in (i) we get

$\left(\frac{a}{2}\right)^2 + y^2 = a^2 \ \Rightarrow y = \frac{\sqrt{3}a}{2}$

$\therefore$ Coordinates of A are $(\frac{a}{2}, \frac{\sqrt{3}a}{2})$

20. $\frac{15}{2}$ sq units (self-practise)

21. a = 1 , b = 0 (self-practise)

22. $\mathrm{BD} : \mathrm{CD} = 1 : 2$

Coordinates of D are $\left(\frac{1\times4+2\times-1}{1+2}, \frac{1\times2+2\times-2}{1+2}\right)$, i. e. $(\frac{2}{3}, \frac{-2}{3})$

$\mathrm{AD} = \sqrt{(\dfrac{2}{3} - 0)^2 + (\dfrac{-2}{3} - 3)^2} = \dfrac{5\sqrt{5}}{3}$ units

$\mathrm{DP} = \mathrm{AD} - \mathrm{AP} = \dfrac{5\sqrt{5}}{3} - \dfrac{2\sqrt{5}}{3} = \sqrt{5}$ units

$\therefore \ \dfrac{\mathrm{AP}}{\mathrm{DP}} = \dfrac{2}{3}$

P divides AD in the ratio $2 : 3$

x-coordinate of P is $= \frac{2\times\frac{2}{3}+3\times0}{2+3} = \frac{4}{15}$

y-coordinate of P is $= \frac{2\times\frac{-2}{3}+3\times3}{2+3} = \frac{23}{15}$

$\therefore$ Coordinates of P are $\left(\dfrac{4}{15}, \dfrac{23}{15}\right)$

23. Let (x, y) be coordinates of D.

Since we know that diagonals of a parallelogram bisect each other.

Mid-point of AC= $(\frac{15}{2}, \frac{5}{2})$

Mid-point of BD= $(\frac{x+8}{2}, \frac{y+2}{2})$

By comparison $\frac{x+8}{2} = \frac{15}{2}, \frac{y+2}{2} = \frac{5}{2}$

$x = 7, y = 3$

coordinates of D are (7,3) and coordinates of E are $\left(8, \frac{7}{2}\right)$

Area of ΔADE

$= \dfrac{1}{2} |[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$

$= \frac{1}{2} |[6\left(3 - \frac{7}{2}\right) + 7\left(\frac{7}{2} - 1\right) + 8(1 - 3)]| = \frac{1}{2}\left|-\frac{3}{2}\right| = \frac{3}{4}$

24.

a. D is the mid-point of BC

So, $(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2})$

b. If P is point on AD, then by section formula

$$\frac{2\left(\frac{x_2+x_3}{2}\right)+1*x_1}{1+2}, \frac{2\left(\frac{y_2+y_3}{2}\right)+1*y_1}{1+2}$$

$$\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}$$

c. Same $\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}$

∴ Its centroid divides all the medians in the ratio 2:1

d. Coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}\right)$ and $\left(\frac{y_1+y_2+y_3}{3}\right)$

25. Since P is the mid point of the line segment A (-10, 4) and B (-2, 0)

The coordinates of the point P are (-6, 2)
Let P divides the joining of C (-9, -4) and D (-4, y)in the ratio k:1
P $\left(\frac{-4k-9}{k+1}, \frac{ky-4}{k+1}\right)$ = (-6, 2)
$\Rightarrow$ $\frac{-4k-9}{k+1}$ = -6 and $\frac{ky-4}{k+1} = 2$ ….. (i)
$\frac{-4k-9}{k+1}$ = -6 therefore k = $\frac{3}{2}$
Ratio is 3:2
From equation (i)
$\frac{ky-4}{k+1}$ = 2 put the value of k we get y = 6

26. Since O is the midpoint of BC and coordinates of C are (0, -3)

Coordinate of B are (0, 3)
Now AO will be the perpendicular bisector of BC.
Therefore, A will lie on x-axis. Let coordinates of A are (x,0)
Now, AB=BC
$\sqrt{(x-0)^2+(0-3)^2}$ = 6
$\sqrt{x^2+9}$ = 6
X = ± $3\sqrt{3}$
Coordinates of A are ($3\sqrt{3}$, 0) or ($-3\sqrt{3}$, 0)
When A is ($3\sqrt{3}$, 0) then D will be ($-3\sqrt{3}$, 0) so that BACD is a rhombus

27. Area of triangle ABC

= 1/2| t(t + 2 - t) + (t + 2)[t - (t - 2)] + (t + 3)[(t - 2) - (t + 2)] = 4 square units which is independent of t

28. Area of $\Delta ABC = \frac{1}{2}|6(5+2)-3(-2-3)+4(3-5)| = \frac{49}{2}$ sq. units

Area of $\Delta PBC = \frac{1}{2}|x(-2-5)+4(5-y)-3(y+2)|$
$= \frac{1}{2}|-7x+20-4y-3y-6| = \frac{1}{2}|-7(x+y-2)| = \frac{7}{2}|x+y-2|$ sq. units
Now, $\frac{Ar(\Delta PBC)}{Ar(\Delta ABC)} = \frac{\frac{7}{2}|x+y-2|}{\frac{49}{2}} = \left|\frac{x+y-2}{7}\right|$
Hence, proved.

CHAPTER-8 – INTRODUCTION TO TRIGONOMETRY

Level-1

1	Value of $\cos 48° - \sin 42°$ (a) 1 (b) -1 (c) 0 (d) none of these
2	If $\sin A = \frac{24}{25}$ then $\cos A =$ (a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) 1 (d)none of these
3	In $\triangle OPQ$, rt $\angle$d at P, OP = 7cm OQ – PQ = 1cm then sin O = (a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) 1 (d)none of these
4	$9\sec^2 A - 9\tan^2 A =$ (a) 1 (b) 9 (c) 8 (d) 0
5	If $\cos \Theta = \frac{1}{2}$ and $\sin \Phi = \frac{1}{2}$ then value of $\Theta + \Phi =$ (a) 30^0 (b) 60^0 (c) 90^0 (d) 120^0
6	Value of $\sin60^0\cos30^0 - \cos60^0\sin30^0$ (a) 1 (b) -1 (c) 0 (d) none of these
7	Sin2A = 2sinAcosA is true when A= (a) 0 (b) 30^0 (C) 45^0 (d) any angle
8	Product of $\tan 1^0 \tan 2^0 \tan 3^0$ - - - - - - - - $\tan 89^0$ is a) 1 (b) -1 (c) 0 (d) none of these
9	Sin A = Cos A is true when (a) 0 (b) 30^0 (C) 45^0 (d) any angle
10	$\triangle$ ABC is isosceles rt $\angle$d at C then value of Cos (A+B) is (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) n.d
11	Find the value of tan 48° tan 23° tan 42° tan 67°
12	If $2\sin 2\theta = \sqrt{3}$, then find the value of θ
13	If sin A=1/2, then find the value ofcos A.
14	find the value of $\cos^2 45° + \sin^2 60°$.
15	Find the value of cos 1°cos2° cos3° ------ cos180°
16	If $\tan\Theta = \frac{a}{b}$,then find the value of $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$
17	If $\sec \alpha = \frac{5}{4}$, evaluate $\frac{1 - \tan\alpha}{1 + \tan\alpha}$
18	What is the maximum value of $\frac{1}{\sec\theta}$
19	If $\cos A = \frac{2}{3}$,then find value of $2\sec^2 A + 2\tan^2 A - 7$

20	Find the value of Tan5°×tan30° × $4tan85°$

Level-2

1	If sec5A = cosec (A+36^0), where 5A is an acute angle find value of A
2	If A,B,C are interior angles of a△ABC ,prove that tan($\frac{C+A}{2}$) =cot$\frac{B}{2}$
3	If √3tan2x =cos 60^0+ sin45^0cos45^0
4	find the value of θ if sin 3θ=cos(θ-6°) where 3 θ, θ-6^0 are acute angle
5	If sinΘ=$\frac{5}{13}$, find the value of $\frac{sin^2\theta-cos^2\theta}{2sin\theta cos\theta}\times\frac{1}{tan^2\theta}$
6	Find the value of tan 60^0 geometrically.
7	Find the value of cosec 30^0 geometrically.
8	Express sin 81^0 + tan71^0 in terms of trigonometric ratios of angles between 0^0 and 45^0.
9	Evaluate ($sin^4Θ$ - $cos^4Θ$) ÷ ($sin^2Θ$- $cos^2Θ$)
10	If sec A –tanA=4 , prove that cos A = $\frac{8}{17}$

Level-3

1	Prove that (1+$tan^2\theta$)(1-sinθ)(1+ sinθ)=1
2	If tanΘ=$\frac{a}{b}$, prove that $\frac{asin\theta-bcos\theta}{asin\theta+bcos\theta}=\frac{a^2-b^2}{a^2+b^2}$
3	if tan A =√2-1 show that sinAcosA = $\frac{\sqrt{2}}{4}$
4	If A+B = 90^0,prove that $\sqrt{\left(\frac{tanAtanB+tanAcotB}{sinAsecB}-\frac{sin^2B}{cos^2A}\right)}$ = tan A
5	If cos(40-Θ) – sin(50-Θ)+ $\frac{cos^2 40°+cos^2 50°}{sin^2 40°+sin^2 50°}$
6	If $7sin^2A+3cos^2A=4$,find the value of secA+cosecA
7	Prove that $\frac{sec^2\theta-sin^2\theta}{tan^2\theta}$ = $cosec^2Θ-cos^2Θ$
8	Evaluate $\frac{sec41°sin49°+cos29°cosec61°-\frac{2}{\sqrt{3}}(tan20°tan60°tan70°)}{3(sin^2 31°+sin^2 59°)}$
9	Prove that $\frac{tanA+tanB}{cotA+cotB}$ = tanAtanB
10	Evaluate $\frac{cos^2 20°+cos^2 70°}{sec^2 50°-cot^2 40°}$ + $2cosec^2 58° - 2cot58°tan32° -$ $4tan13°tan37°tan45°tan53°tan77°$

Level-4

1	sec θ+tan θ=m and sec θ-tan θ=n then prove that mn =1.
2	If aCosΘ - bCosΘ = x , aSinΘ+ bCosΘ= y ,

	then prove that $a^2 + b^2 = x^2 + y^2$
3	Solve for Θ if $\frac{cos^2\theta - 3cos\theta + 2}{sin^2\theta} = 1$
4	Solve for θ for $0° < \theta \leq 90°$, $3\tan\theta + \cot\theta = 5$ cosec θ
5	if $\sin\theta + \sin^2\theta = 1$,prove that $\cos^2\theta + \cos^4\theta = 1$
6	If $\tan\Theta = \frac{1}{\sqrt{7}}$,find the value of $\frac{cosec^2\theta - sec^2\theta}{cosec^2\theta + sec^2\theta}$
7	If cosec θ- sinθ=a ,secθ-cosθ = b. Prove that $a^2b^2(a^2 + b^2 + 3) = 1$
8	$prove\ that\ (\frac{1}{sec^2\theta - cos^2\theta} + \frac{1}{cosec^2\theta - sin^2\theta})\sin^2\theta cos^2\theta = \frac{1 - sin^2\theta cos^2\theta}{2 + sin^2\theta cos^2\theta}$
9	Evaluate $\frac{4}{3}\tan^2 30° + \sin^2 60° - 3\cos^2 60° + \frac{3}{4}\tan^2 60° - 2\tan^2 45°$
10	Prove that $\frac{sin\theta}{cot\theta + cosec\theta} = 2 + \frac{sin\theta}{cot\theta - cosec\theta}$

ANSWERS

LEVEL 1

1	(C)
2	(A)
3	(B)
4	(B)
5	(C)
6	(D)
7	(D)
8	(A)
9	(C)
10	(A)
11	1
12	30^0
13	3/4
14	2
15	0

16	$\frac{b+a}{b-a}$
17	1/7
18	1
19	0
20	$4/\sqrt{3}$

Level 2

1	9
3	$X= 15^{0}$
5	$\frac{595}{3456}$
9	1

Level3

5	1
6	$2(1+\sqrt{3}) / \sqrt{3}$
8	0
10	-1

Level 4

3	$\Theta= 0$
4	$\Theta= 60^{0}$, $0<\Theta<90$
6	3/4
9	25/36

CHAPTER -9 :Applications of Trigonometry

Subject Enrichment Material for High Achievers

Section – A : M.C.Qs

Choose the correct option

Q.1. A pole 6m high costs a shadow $2\sqrt{3}$ long on the ground, then sun's elevation is :

(a) 60^O (b) 45^O (c) 30^O (d) 90^O

Q.2. IF the length of the shadow of a tower is increasing, then the angle of elevation of the sun will

(a) increase (b) decrease (c) remains same

(d) none of above

Q.3. If the length of a tower and the distance of the point of observation from its foot, both are increased by 10%, then the angle of elevation of its top

(a) increase (b) decrease (c) remains same

(d) none of above

Q.4. Two poles are 'a' metres apart and the height of one is double of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the smaller pole is

(a) $\sqrt{2a}$ metres (b) $\frac{a}{2\sqrt{2}}$ metres (c) $\frac{a}{\sqrt{2}}$ (d) 2*a* metres

Q.5. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$. The angle of elevation of the sum is

(a) 30^O (b) 45^O (c) 60^O (d) 90^O

Section – B : 3 or 4 Mark Questions

Q.1. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angels of elevation of the bottom and the top of the flag-staff are α and β respectively. Prove that the height of the tower is $\frac{h\tan\alpha}{\tan\beta-\tan\alpha}$.

Q.2. The angle of elevation of the top of a vertical tower from a point P on the ground is 60^O. From another point Q, 10m vertically above P, the angle of elevation of the top of the tower is 45^O. Find the height of the tower.

Q.3. A window of a house is h metres above the ground. From the window, the angles of elevation and depression of the top and the bottom of another house situated on the opposite side of the lane are found to be α and β, respectively. Prove that the height of the other house is *h (1 + tan* α *tan* β*) metres.*

Q.4. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 100 metres from the tower. After 10 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water?

Q.5. Two persons are on either sides of a tower of height 50 m. The persons observes the top of the tower at an angle of elevation of 30° and 60°. If a car crosses these two persons in 10 seconds, what is the speed of the car?

Q.6. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and he is 5 m away from the wall, what is the length of the window?

Q.7. Two ships are approaching a light-house from opposite directions. The angles of depression of the two ships from the top of the light-house are 30^O and 45^O. If the distance between the two ships is 100m, find the height of the light-house. [Use $\sqrt{3} = 1.732$]

Q.8. The angle of elevation of a jet plane from a point A on the ground is 60^O. After a flight of 15 seconds the angle of elevation changes to 30^O. If the jet plane is flying at a constant height of $1500\sqrt{3}m$, find the speed of the jet plane.

Q.9. A bird is sitting on the top of a 80m high tree. From a point on the ground, the angle of elevation of the bird is 45^O. the bird flies away horizontally in such a way that it remained at the constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30^O. Find the speed of flying of the bird. (Take $\sqrt{3} = 1.732$).

Q.10. A man standing of the deck of a ship, which is 10m above water level, observes the angle of elevation of the top of a hill as 60^O and the angle depression of the base of hill as 30^O. Find the distance of the hill froim the ship and the height of the hill.

Q.11. The angle of elevation of a cloud from a point 60m above the lake is 30^{O} and the angle of depression of its reflection of the cloud in water is 60^{O}. Find the height of the cloud.

Q.12. If the angel of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is $\frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$.

Q13. From the foot and the top of a building of height 230 m, a person observes the top of a tower with angles of elevation of b and a respectively. What is the distance between the top of these buildings if tan a = 5/12 and tan b = 4/5

Q14. A boy standing on a horizontal plane finds a bird flying at a distance of 100m from him at an elevation of 300. A girl standing on the roof of 20 meter high building finds the angle of elevation of the same bird to be 450. Both the boy and the girl are on opposite sides of the bird. Find the distance of the bird from the girl.

Q15 The angle of elevation of top of tower from two points at a distance a metre and b metre from the base of the tower and in the same straight line with it are complementary. Find the height of the tower

Answers

Section – A (MCQ's)

1. (a)
2. (b)
3. (c)
4. (b)
5. (a)

Section – B

1. $5\sqrt{3}\left(\sqrt{3}+1\right)$ meter
2. 26.28 km/hr
3. $24\sqrt{3}$ km/hr
4. $5\left(\sqrt{3}-1\right)^{m}$
5. 36.6 metres
6. 720 km/hr
7. $\dfrac{80\left(\sqrt{3}-1\right)}{2} m/\sec$
8. Distance of the hill from the ship=$10\sqrt{3}m$, Height of the hill = 40m.
9. 120m

13 650n

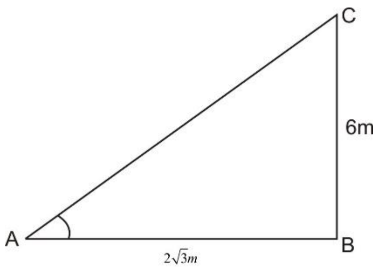
14 = 30 $\sqrt{2}$ m

15 √ab m

ANSWERS

Section – A

Sol.1. Now in right angled ΔABC,

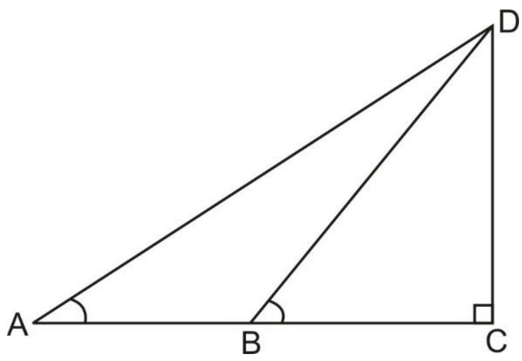


Tan $A = \frac{6}{2\sqrt{3}} = \sqrt{3}$

$\therefore \ \angle A = 60^O$

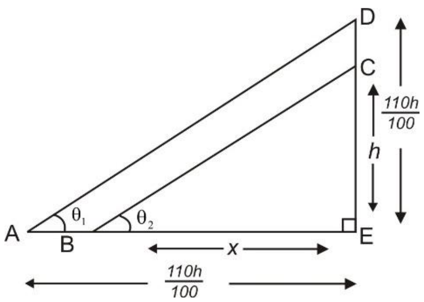
Hence correct alternative = (A).

Sol.2.



False. We observe that if the length of the shadow of a tower is increasing, then the angle of elevation of the sun is decreasing.

Sol.3. When both lengths of the tower and distance of observation from its foot are increased by 10%, then we get the adjacent figure.



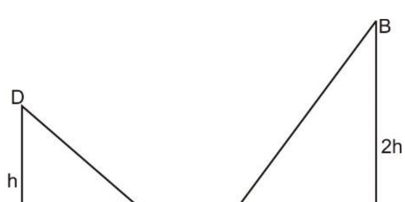
$$\tan\theta_1 = \frac{h}{x}$$

$$\tan\theta_2 = \frac{h}{x}$$

$$\therefore \ \ \theta_1 = \theta_2$$

Which shows that AD || BC and angle of elevations are equal. Hence the statement is true.

Sol.4. BC & DE are towers, A is point of observation.



$\tan\theta$ = $\dfrac{2h}{a/2}$ ....(1)

tan (90-θ) = $\dfrac{h}{a/2}$

Cot θ = $\dfrac{h}{a/2}$ ....(2)

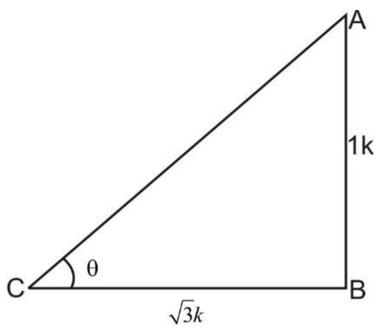
Multiply (1) & (2)

tan θ x Cotθ = $\dfrac{2h}{a/2}$ X $\dfrac{h}{a/2}$

1 = $\dfrac{2h^2}{(a/2)^2}$ or h^2 = $\left(\dfrac{a}{2}\right)^2 \times \dfrac{1}{2}$

$$h = \frac{a}{2\sqrt{2}}$$

Sol.5.



AB= length of Rod

BC = length of shadow

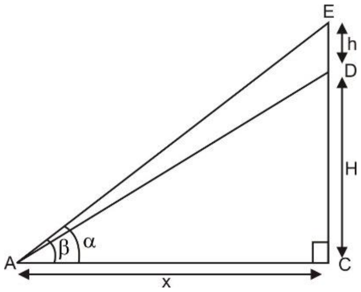
Elevation of sun = θ

$$\tan\theta = \frac{1k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$$

$\therefore \qquad \theta = 30^O$

Section – B

Sol.1. let CD = H metre be the height of the tower and DE = h metre be the height of the flagstaff, which is standing on the tower.



Let A be a point on the plane, which makes an angle of β and α to the top and bottom of the flag.

Also, let AC = *x m*

In right angle ΔCAE,

$$\tan\beta = \frac{CE}{AC} = \frac{CD + DE}{x}$$

$$\Rightarrow \tan\beta = \frac{H + h}{x} \quad .....(i)$$

In right angled ΔCAD,

$$\tan\alpha = \frac{CD}{AC}$$

$$\Rightarrow \tan\alpha = \frac{H}{x}$$

$$\Rightarrow x = H\cot\alpha$$

Putting x = H cot α in Eq. (i), we get

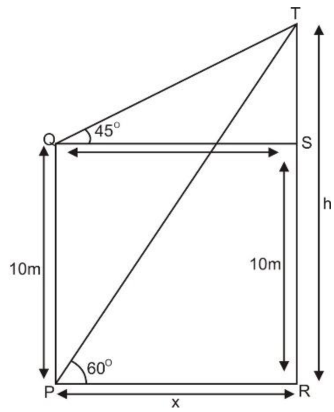
$$\tan\beta = \frac{H + h}{H\cot\alpha}$$

$$= \frac{(H + h)}{H}\tan\alpha$$

$$\Rightarrow H(\tan\beta - \tan\alpha) = h\tan\alpha$$

$$H = \frac{h \tan\alpha}{\tan\beta - \tan\alpha}$$. Hence proved.

Sol.2 Let RT be the height of the tower and let RT = *hm*.



Let PR = *x m*

In ΔQST,

$$\tan 45^o = \frac{h-10}{x}$$

$$\Rightarrow x = h - 10m.$$

Now in ΔPRT,

$$\tan 60^o = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{h-10}$$

$$\Rightarrow \sqrt{3}(h-10) = h$$

$$\Rightarrow \sqrt{3}h - 10\sqrt{3} = h$$

$$\Rightarrow (\sqrt{3}-1)h = 10\sqrt{3}$$

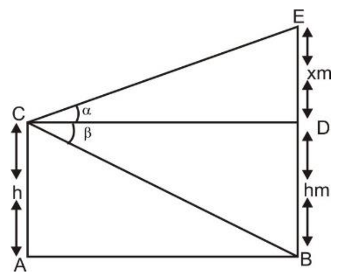
$$\Rightarrow h = \frac{10\sqrt{3}}{\left(\sqrt{3}-1\right)} \times \left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$$

$$\Rightarrow h = \frac{10\sqrt{3}\left(\sqrt{3}+1\right)}{3-1}$$

$$\Rightarrow h = 5\sqrt{3}\left(\sqrt{3}+1\right)$$

Hence height of the tower = $5\left(3+\sqrt{3}\right)m$.

Sol.3. Let C be the position of a window of house AC which is h metres above the ground, i.e., AC = h m. BE be the house on the opposite side of the street. The angel of elevation and depression of the top and foot of the opposite house from the window C be α and β respectively.



i.e., $\angle DCE = \alpha$

and $\angle BCD = \beta$

Let DE = x m

In right triangle CDE, we have

$$\tan\alpha = \frac{DE}{CD}$$

$$\Rightarrow \quad \tan\alpha = \frac{x}{CD}$$

$$\Rightarrow \quad CD = \frac{x}{\tan\alpha}$$

$$\Rightarrow \quad CD = x\cot\alpha$$

In right triangle BCD, we have

$$\tan \beta = \frac{BD}{CD}$$

$$\Rightarrow \quad CD = \frac{BD}{\tan \beta}$$

$$\Rightarrow \quad CD = \frac{h}{\tan \beta}$$

$\Rightarrow$ CD = h cot β

Comparing (i) and (ii), we get

x cot α = h cot β

$$\Rightarrow \quad x = \frac{h \cot \beta}{\cot \alpha}$$

$\Rightarrow$ x = h cot β . tan α

Hence, height of the opposite house (BE)

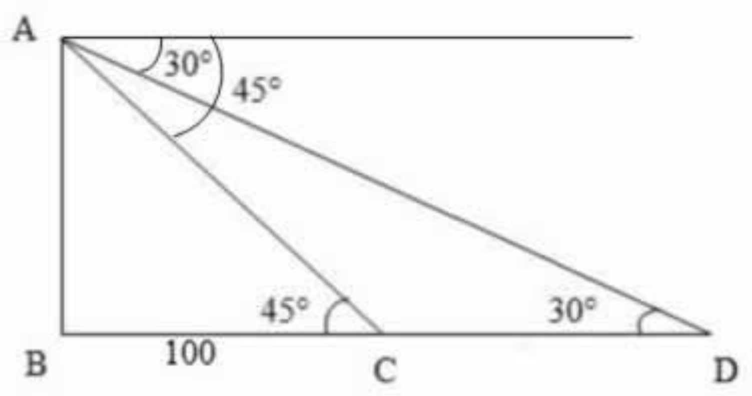
$$= BD + DE$$

$$= h + x$$

$$= h + h \cot \beta \, . \tan \alpha$$

$$= h(1 + \cot \beta \, . \tan \alpha).$$

Sol 4.



Let AB be the tower. Let C and D be the positions of the boat

$\angle ACB = 45°$, $\angle ADC = 30°$, BC = 100 m

tan 45° = AB/BC =>1=AB/100 => AB = 100 ------(1)

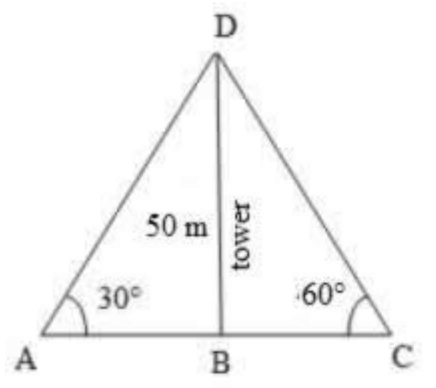
tan 30° = AB/BD =>1/√3=100/BD (∵ Substituted the value of AB from equation 1)

=> BD = $100\sqrt{3}$ CD = (BD - BC) = (100√3−100)=100(√3−1)

given that the distance CD is covered in 10 seconds.

Required speed = Distance / Time=10(1.73−1)=7.3 m/s=7.3×3.6 km/hr=26.28 km/hr

Sol. 5.



Let BD be the tower and A and C be the positions of the persons.

Given that BD = 50 m, $\angle$ BAD = 30° , $\angle$ BCD = 60°

in right $\triangle$ ABD,

tan 30° = BD/BA ⇒ 1/√3=50/BA ⇒BA=50√3

tan 60° = BD/BC ⇒√3=50 /BC ⇒BC=50 / √3

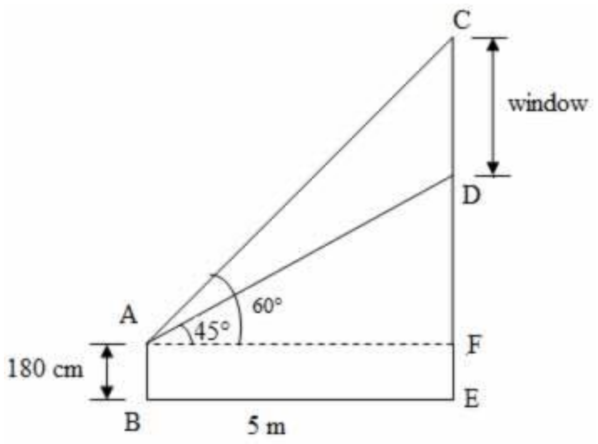
Distance between the two persons = AC = BA + BC

=50√3+50/√3=200/√3 m

the distance travelled by the car in 10 seconds = 200/√3 m

Speed of the car = Distance / Time=20/√3 m/s=20/√3×3.6 km/hr=24√3 km/hr

Sol.6



Let AB be the man and CD be the window

the height of the man, AB = 180 cm, the distance between the man and the wall, BE = 5 m,

$\angle$DAF = 45° , $\angle$CAF = 60°

AF = BE = 5 m

In right $\triangle$ AFD,

tan45°=DF/AF   1=DF/5   DF = 5------ (1)

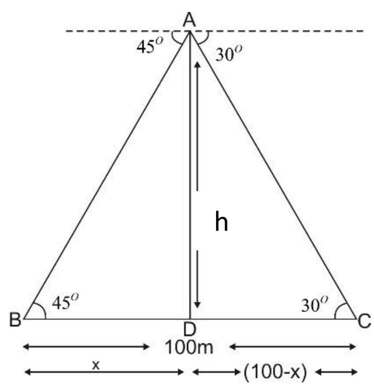
In right $\triangle$ AFC,

tan60°=CF/AF   √3=CF/5   CF=5√3------ (2)

Length of the window = CD = (CF - DF)

$$= 5\sqrt{3} - 5 = 5\left(\sqrt{3} - 1\right)$$

Sol 7. Let AD be the light house and its height be h . The distance of one ship from the lght house is x and that of other ship is 100- x .



In ΔABD,

$\tan 45^0 = \dfrac{AD}{BD}$

$1 = \dfrac{h}{x}$

$h=x$ ......(1)

In ΔADC,

$\tan 30^0 = \dfrac{AD}{CD}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{100-x}$

$100 - h = \sqrt{3}h$

$\sqrt{3}h + h = 100$

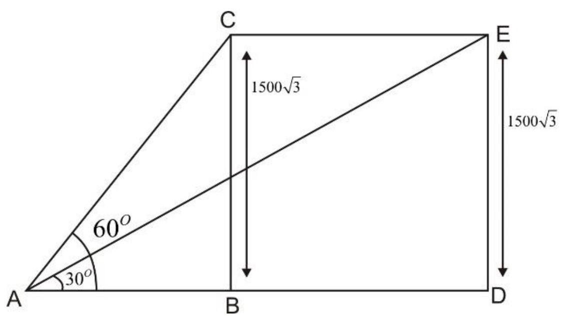
$h(\sqrt{3}+1) = 100$

$h = \dfrac{100}{\sqrt{3}+1} = \dfrac{100}{\sqrt{3}+1} \text{ X } \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= 50 \text{ X}(1.732-1)$

$h = 36.6$m.

Sol 8. Let A be the point of observation, C and E be the two points of the plant it is given that after 15 seconds angle of elevation changes from 60^0 to 30^0.



i.e., $\angle$BAC = 60^0 and $\angle$DAE = 30^0, It is also given that height of the jet plane is $1500\sqrt{3}$ m.

i.e. CB = 1500 $\sqrt{3}$.

(Since jet plane is flying at constant height, therefore, CB = ED = $1500\sqrt{3}$ m

In right triangle ABC, we have

$$\tan 60^O = \frac{BC}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{1500\sqrt{3}}{AB}$$

$$\Rightarrow \quad AB = \frac{1500\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \quad AB = 1500m$$

In right triangle ADE, we have

$$\tan 30^O = \frac{DE}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{DE}{AB+BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AB+BD}$$

$\Rightarrow$ AB + BD = $1500\sqrt{3}\times\sqrt{3}$

$\Rightarrow$ AB + BD = 4500

Putting the value of (i) in (ii), we get

1500 + BD = 4500

BD = 3000

$\therefore$ Distance traveled in 15 sec

= CE = BD = 3000 metres.

Now, speed of plane (m/s) $= \frac{3000}{15} = 200 m/s$

And, speed of plane (km/h) $= \frac{200}{1000} \times 3600$

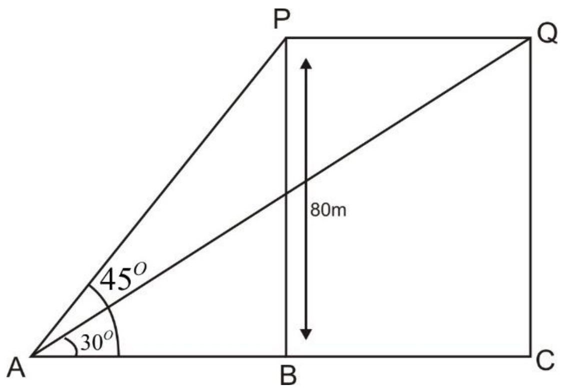
= 720 km/hr.

Sol.9. Let P and Q be the two positions of the bird, and Let A be the point of observations. Let ABC be the horizontal line through A.

Given ; The angles of elevations of that bird in two positions P and Q from point A are 45^O and 30^O, respectively.

$\therefore$ $\angle PAB = 45^O$

And $\angle QAB = 30^O$



Also, PB = 80m

In $\Delta ABP, we$

Have : $\tan 45^O = \frac{BP}{AB}$

$\Rightarrow \quad 1 = \frac{80}{AB}$

$\Rightarrow$ AB = $80m$

In ΔQAC,

$$\tan 30^O = \frac{CQ}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{AC}$$

$$AC = 80\sqrt{3}$$

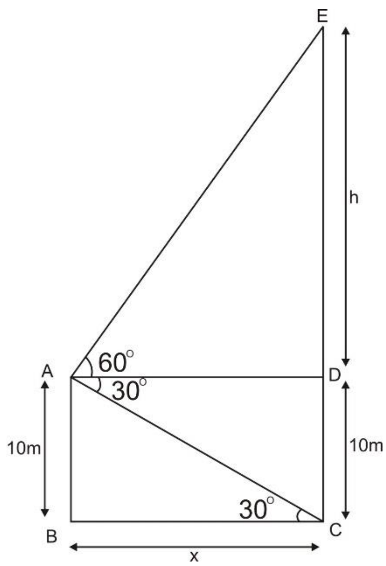
$$PQ = BC = AC - AB$$

$$= 80\sqrt{3} - 80$$

$$= 80\left(\sqrt{3} - 1\right)$$

$$Speed = \frac{Dis\tan ce}{Time} = \frac{80\left(\sqrt{3}-1\right)}{2} = 40\left(\sqrt{3}-1\right) m/\sec$$

Sol.10. Suppose the man is standing on the deck of the ship at point A. Let CE be the hill with base at C.



Suppose ED= h *m* and BC = *x m*

In ΔEAD,

We have $\tan 60^O = \frac{ED}{AD}$

$\Rightarrow \sqrt{3} = \frac{h}{x}(AD = BC = x)$

$\Rightarrow \quad h = x\sqrt{3}$ ....(i)

In ΔABC,

We have $\tan 30^{O} = \frac{AB}{BC}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{x}$

$\Rightarrow \quad x = 10\sqrt{3}$ ....(ii)

$\therefore$ Distance of the hill from the ship $10\sqrt{3}$ m.

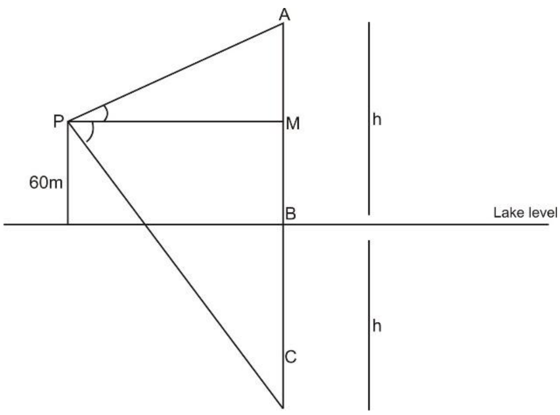
From (i) and (ii), we have

$$h = 10\sqrt{3} \times \sqrt{3} = 30m$$

$\therefore$ Height of the hill = $h + 10$

$$= 30 + 10 = 40m$$

Sol .11



Let AB – m Height of Cloud = h (say)

BC- depth of Reflection = h

AB = BC (by laws of reflection)

In Δ APM

tan 30^0 = $\dfrac{Am}{PM}$

tan 30^0 = $\dfrac{h-60}{PM}$

PM = $\dfrac{h-60}{\tan 30^0}$

In Δ CMP

tan 60^0 = $\dfrac{h+60}{PM}$

From 1+2

$\dfrac{h+60}{\tan 60}$ = $\dfrac{h-60}{\tan 30^0}$

tan 30^0 $(h+60)$ = tan (h-60^0)

$\dfrac{(h+60)}{\sqrt{3}}$ = $\sqrt{3}\ (h-60)$

h + 6 0 = 3 $(h-60)$

h + 60 = 3h -180

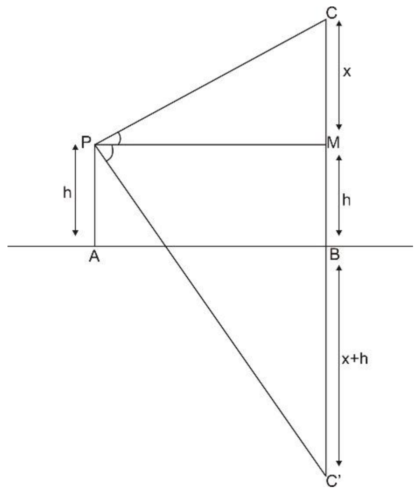
60+180 = 3h - h

240 = 2h

h = $\dfrac{240}{2}$ = 120

Height of cloud = 120m

Sol.12. Let AB is the surface of the lake and let P be a point of observation such that AP = h metres. Let C be the position of the cloud and C' be its reflection in the lake. Then CB=C'B (Laws of Reflection)



Let CM = *x*.

In ΔCPM, we have

$$\tan\alpha = \frac{CM}{PM}$$

$$\Rightarrow \quad \tan\alpha = \frac{x}{AB}$$

$\Rightarrow$ AB = x cot α ....(i)

In ΔPMC, we have

$$\tan\beta = \frac{C'M}{PM}$$

$$\Rightarrow \quad \tan\beta = \frac{x+2h}{AB}$$

$\Rightarrow \quad AB = (x+2h)\cot\beta$ ....(ii)

From (i) and (ii), we have

$$x\cot\alpha = (x+2h)\cot\beta$$

$$\Rightarrow \quad x(\cot\alpha - \cot\beta) = 2\,h\ \cot\ \beta$$

$$\Rightarrow \quad x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow \quad x\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow \qquad x = \frac{2h \tan \alpha}{\tan \beta - \tan \alpha}$$

Height of cloud = h + x

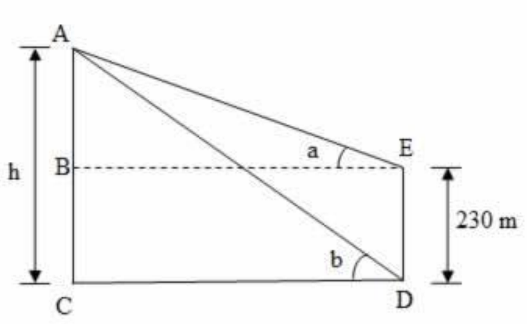
$$= h + \frac{2h \tan \alpha}{\tan \beta - \tan \alpha}$$

$$= h\left[1 + \frac{2\tan \alpha}{\tan \beta - \tan \alpha}\right]$$

$$= h\left[\frac{\tan \beta - \tan \alpha + 2\tan \alpha}{\tan \beta - \tan \alpha}\right]$$

$$= h\left(\frac{\tan \beta + \tan \alpha}{\tan \beta - \tan \alpha}\right)$$. Hence proved.

Sol.13.



Let ED be the building and AC be the tower.

Given that ED = 230 m, ∠ADC = b, ∠AEB = a

given that tan a = 5/12 and tan b = 4/5

Let AC = h

Required Distance = Distance between the top of these buildings = AE

in right △ ABE,

tan(a)=AB/BE =>5/12=(h-230)/BE

=>BE =12(h-230)/5------ (1)

in right △ ACD,

tan(b)=AC/CD =>4/5=h/CD

=>CD = 5h/4------ (2)

now BE = CD

=>12(h-230)/5=5h/4 from (1) & (2)

=>48h−(4×12×230)=25h

=>23h=(4×12×230)

=>h=(4×12×230)/23=480 m ------ (3)

in right △ ABE

AB = (AC - BC)

= (480 - 230)

= 250 m

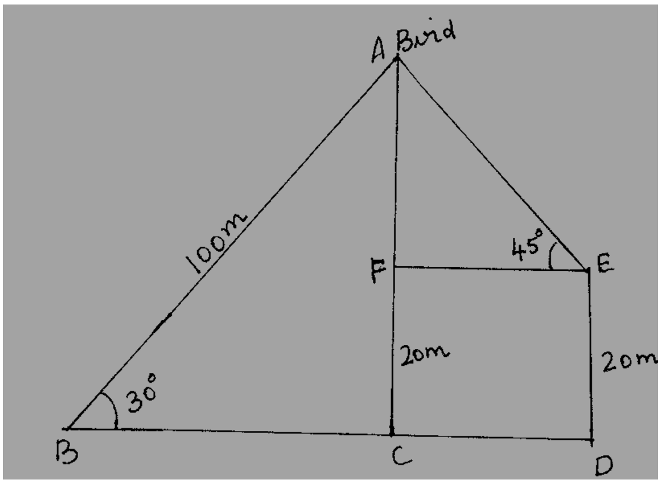
tan(a) = 5/12. Firstly find sin(a) now by using pyth.thm

sin(a) = 5/13=>AB/AE=5/13=>AE = AB×13/5=250×13/5=650 m

i.e., Distance between the top of the buildings = 650 m

Sol.14.

In right Δ ACB



$$\text{Sin } 30^0 = \frac{AC}{AB}$$

$$\frac{1}{2} = \frac{AC}{100}$$

2 AC = 100

AC = 50m

$\Rightarrow$ AF = (50 – 20) = 30m

In right Δ AFE

$$\text{Sin } 45^0 = \frac{AF}{AE}$$

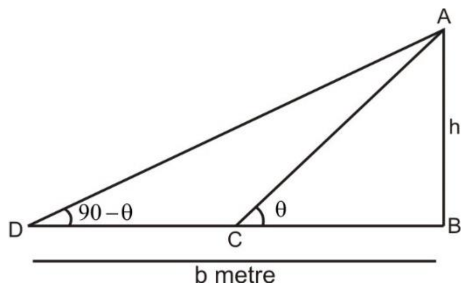
$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{30}{AE}$$

AE = 30 $\sqrt{2}$

= 30 x 1.414

= 42.42m

Sol. 15.



Let AB is height of tower = hm

C&D point of observations

In Δ ABC

tan θ - $\frac{h}{a}$

In Δ ABD

Tan (90-θ) = $\frac{h}{b}$

Cot θ = $\frac{h}{b}$

Multiply 1+2

Tan θ x cot $\theta = \frac{h}{a} \times \frac{h}{b}$

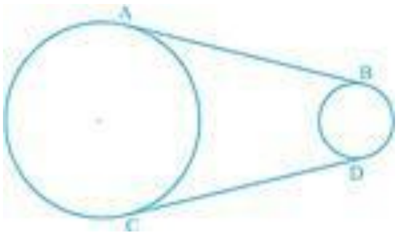
$1 = \frac{h^2}{ab}$

$h = \sqrt{ab}$

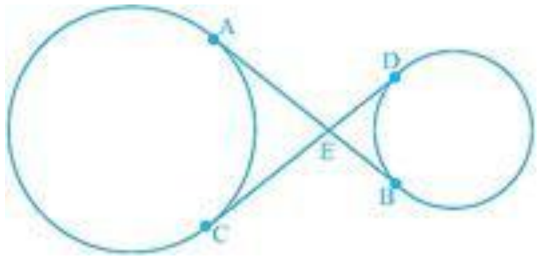
CHAPTER-10 –CIRCLES

Q1	From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Find the area of the quadrilateral PQOR
Q2	At one end A of diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. Find the length of the chord CD parallel to XY and at a distance 8 cm from A .
Q3	If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm.Find the length of each tangent
Q4	If from an external point B of a circle with centre 'O'., two tangents BC, BD are drawn such that ∠DBC = 120°, prove that BC + BD = BO, i.e., BO = 2BC
Q5	Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines

Q6 **In the given figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.**



7. **In the given figure, common tangent AB and CD to two circles intersect at E. Prove that AB = CD.**



8. **A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.**

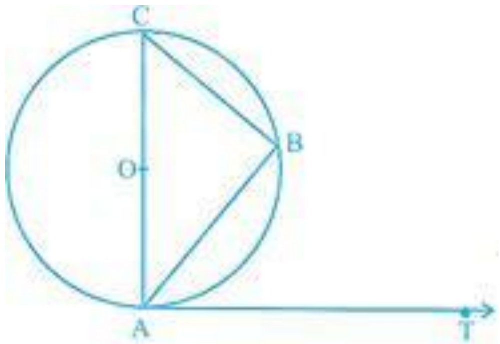
9. **If a hexagon ABCDEF circumscribe a circle,**

then prove that AB + CD + EF = BC + DE +

FA

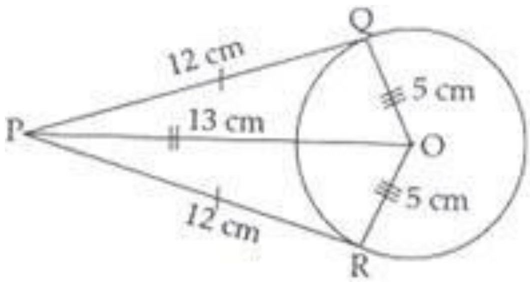
10 From an external point P, two tangents PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D respectively. If PA = 10 cm, find the perimeter of ΔPCD

11 If AB is a chord of a circle with centre O. AOC is a diameter and AT is the tangent at A as shown in figure. Prove that ∠BAT = ∠ACB



Sol. 1): PQ is tangent and QO is radius at contact point Q.

∴ ∠PQO = 90°



∴By Pythagoras

theorem, $PQ^2 = OP^2 -$

OQ^2

$= 13^2 - 5^2 = 169 - 25 = 144$

⇒PQ = 12 cm

∴ΔOPQ ≅ ΔOPR [By SSS criterion of congruence]

∴Area of ΔOPQ = ar ΔOPR

= 2 × base × altitude

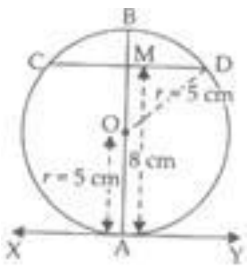
= RP × OR = 12 × 5 = 60 cm^2

Sol.2): XAY is tangent and AO is radius at contact point A of circle.

AO = 5 cm

∴ ∠OAY = 90°

CD is another chord at distance (perpendicular) of 8 cm from A and CMD || XAY meets AB at M.



Join OD.

OD = 5 cm

OM = 8 – 5 = 3 cm

∠OMD = ∠OAY = 90°

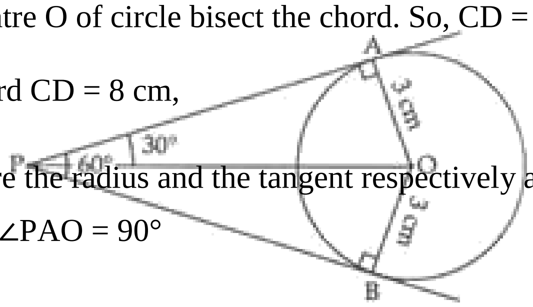
Now, in right angled ΔOMD

$MD^2 = OD^2 - MO^2 = 5^2 - 3^2 = 25 - 9 = 16$

⇒ MD = 4 cm

Perpendicular from centre O of circle bisect the chord. So, CD = 2MD = 2 × 4 = 8 cm.

Hence, length of chord CD = 8 cm,



Sol 3 ∵ OA and PA are the radius and the tangent respectively at contact point A of a circle of radius OA = 3 cm. So, ∠PAO = 90°

In right angled ΔPOA,

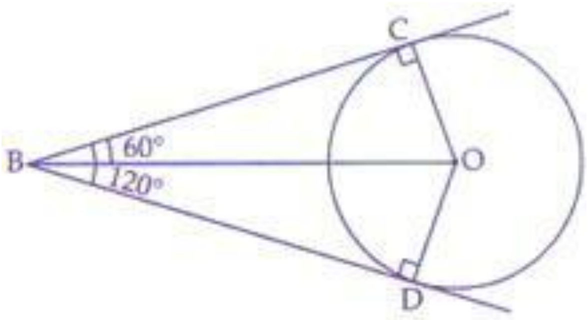
$\tan 30° = \frac{OA}{PA}$ $\frac{1}{\sqrt{3}} = \frac{3}{PA}$

PA= $3\sqrt{3}$

Sol4. Given: A circle with centre O.

Tangents BC and BD are drawn from an external point B such that ∠DBC = 120° **To prove:** BC + BD = BO, i.e., BO = 2BC

Construction: Join OB, OC and OD



Proof: In ΔOBC and ΔOBD, we have

OB = OB [Common]

OC = OD [Radii of same circle]

BC = BD [Tangents from an external point are equal in length] …(i)

∴ ΔOBC ≅ ΔOBD [By SSS criterion of congruence]

⇒ ∠OBC = ∠OBD (CPCT)
∴ ∠OBC = ∠DBC = × 120° [∵ ∠CBD = 120° given]

∠OBC = 60°

OC and BC are radius and tangent respectively at contact point C.

So, ∠OCB = 90°

Now, in right angle ΔOCB, ∠OBC = 60°

cos 60° = $\frac{1}{2} = \frac{BC}{BO}$

⇒ OB = 2BC

⇒ OB = BC + BC

OB = BC + BD [∵ BC = BD

Sol.5 Given: Two intersecting lines AT and BT intersect at T.

A circle with center O touches the above lines at A and B.

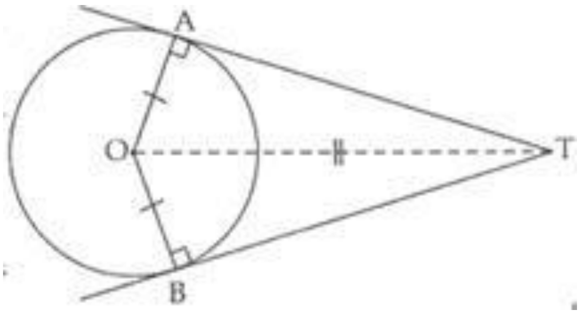
To prove: OT bisects the ∠ATB.

Construction: Join OA and OB.

Proof: OA is radius and AT is tangent at A.

∴ ∠OAT = 90°

Similarly, ∠OBT = 90°



In ΔOTA and ΔOTB, we have

∠OAT = ∠OBT = 90°

OT = OT [Common]
OA = OB [Radii of same circle

∴ ΔOTA ≅ ΔOTB [By RHS criterion of congruence]

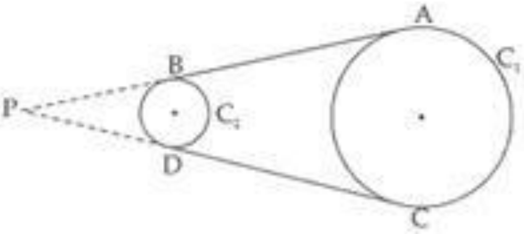
⇒ ∠OTA = ∠OTB [CPCT]

⇒ Centre of circle 'O' lies on the angle bisector of

∠ATB. Hence, proved.

Sol 6. Given: Circles C_1 and C_2 of radii r_1 and r_2 respectively and $r_1 < r_2$.
AB and CD are two common tangents.
To prove: AB = CD

Construction: Produce AB and CD upto P where both tangents meet.



Proof: Tangents from an external point to a circle are equal.

For circle C_1, PB = PD …(i)

and the circle C_2, PA = PC …(ii)

Subtracting (i) from (ii), we have
PA – PB = PC – PD,AB=CD

Sol 7. Given: Two non-intersecting circles are shown in the figure. Two intersecting tangents AB and CD intersect at E. E point is between the circles and outside also.

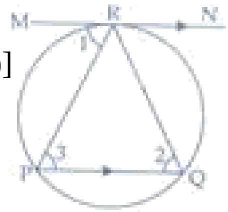
To prove: AB = CD

Proof: We know that tangents drawn from an external point (E) to a circle are equal. Point E is outside of both the circles.

So, EA = EC …(i)

EB = ED …(ii)

⇒ EA + EB = EC + ED [Adding (i) and (ii)]



⇒ AB = CD

Sol.8 Given: In a circle a chord PQ and a tangent MRN at R such that QP || MRN

To prove: R bisects the arc PRQ

Construction: Join RP and RQ

Proof: Chord RP subtends ∠1 with tangent MN and ∠2 in alternates segment of circle
so ∠1=∠2. MRN || PQ

∴ ∠1 = ∠3 [Alternate interior angles]

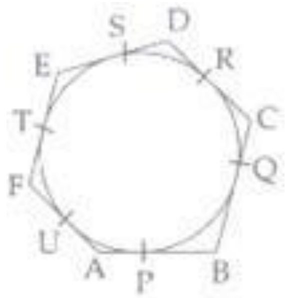
⇒ ∠2 = ∠3

⇒ PR = RQ [Sides opp. to equal ∠s in ΔRPQ]

Equal chords subtend equal arcs in a circle so arc PR = arc RQ or R bisect the arc PRQ

Sol 9. **Given:** A circle inscribed in a hexagon ABCDEF

Sides, AB, BC, CD, DE and DF touches the circle at P, Q, R, S, T and U



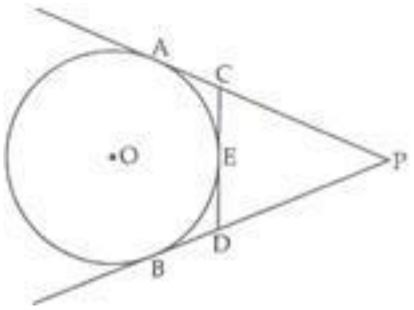
respectively. **To prove:** AB + CD + EF = BC + DE + FA

Proof: We know that tangents from an external F point to a circle are equal.

Here, vertices of hexagon are outside the circle so AP = AU

BP = BQ
CQ = CR
DR = DS
ES = ET
FT = FU
LHS = AB + CD + EF = (AP + BP) + (DR + CR) + (ET + TF)
By using above results, we have
LHS = AB + CD + EF = AU + BQ + DS + CQ + ES + FU
= AU + FU + BQ + CQ + DS + ES
= AF + BC + DE.

Sol.10 Given: A circle with centre O. PA, PB are tangents from an external point P. A tangent CD at E intersect AP and PB at C and D respectively



To find: Perimeter of ΔPCD.
Method: Tangents drawn from an external point to a circle are equal.

$PA = PB = 10$ cm [Given] $CA=CE$

$DB=DE$

Perimeter of $\Delta PCD = PC + PD + CD$
$= PC + PD + CE + DE$
$= PC + CE + PD + DE$
$= PC + CA + PD + DB$
$= PA + PB$
$= 10+10$
$= 20$cm
SOL 11 Given: Chord AB, diameter AOC and tangents at A of a circle with centre O

To prove: $\angle BAT = \angle ACB$

Proof: Radius OA and tangent AT at A are perpendicular

$\therefore \angle OAT = 90°$ (radius at the point of contact of tangent is perpendicular)
$\angle BAT = 90° - \angle BAC$...(i)
AOC is diameter

$\therefore \angle B = 90°$
$\angle C + \angle BAC = 90°$

$\angle C = 90° - \angle BAC$ …(ii)
From (i) and (ii), we get
$\angle BAT = \angle ACB$. Hence, proved

CHAPTER-11: CONSTRUCTIONS

- **Division of a line segment in a given ratio (internally).**
- **Tangents to a circle from a point outside it.**
- **Construction of a triangle similar to a given triangle.**

IMPORTANT QUESTIONS FROM CHAPTER

1 Draw a pair of tangents to the circle of radius 4.5 cm, which are inclined to an angle of 45 degree.

2 Draw a triangle ABC with sides AB = 6 CM, BC = 7.5 cm and AC =6.6 cm. Construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of triangle ABC.

3 Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to similar circle from a point on the largest circle. Also measure its length.

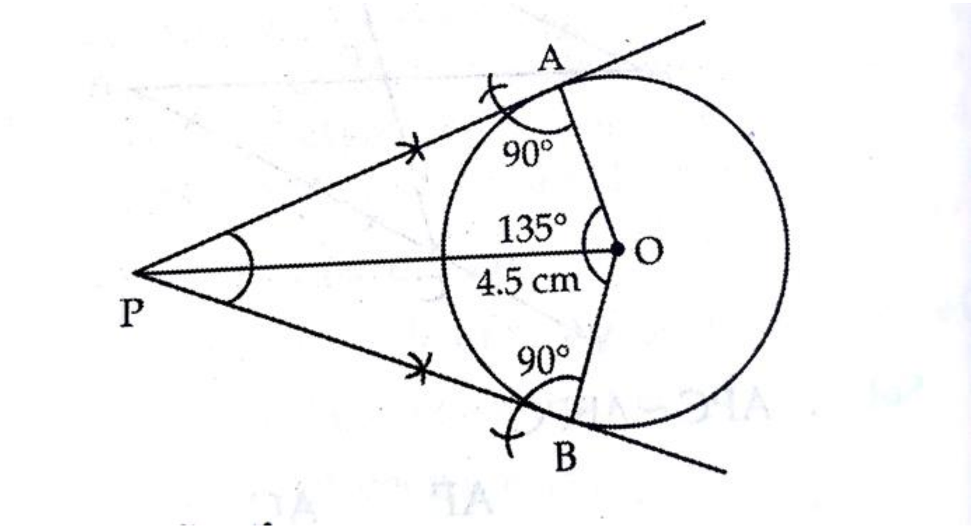
4 Given a rhombus ABCD in which AB = 4 cm and angle ABC= 60°. Divide it into two triangles, ABC and ADC. construct a triangle ABC similar to triangle ABC with scale factor $\frac{2}{3}$. Draw a line segment CD parallel to CD where D lies on AD. Is ABCD a rhombus? give reasons?

5 ABC be a right angle triangle in which AB = 6 CM, BC = 8 cm and angle B = 90°. BD is perpendicular from B on AC. The circle through B, C, D is drawn. Construct tangent from A to this circle.

6 Draw a line segment OB = 11 cm. Take a point P on it, such that OP = 4 cm. Draw the circle with centre P and radius 1.5 cm. Draw the circle with centre B and radius 3.5 cm. Draw the tangent from the point O to the circle with centre P and the circle with centre B. Find the length of the tangent segments.

ANSWERS

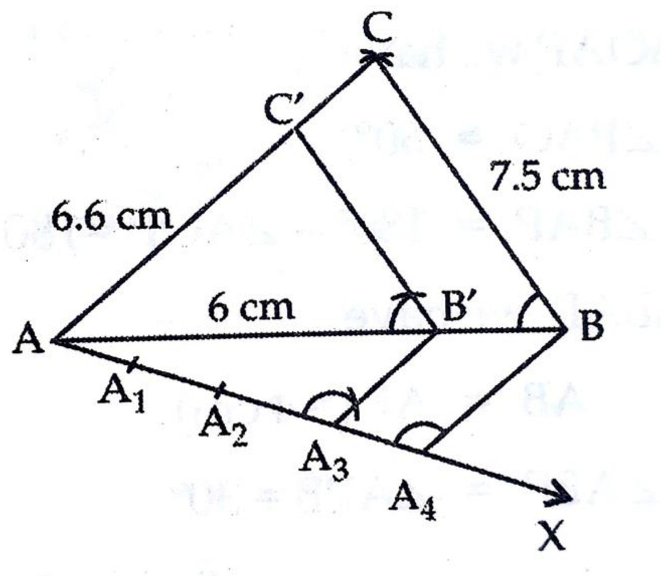
1



Steps of Constructions

a. Draw a circle of radius 4.5 CM.
b. Join OA .
c. Construct angle AOB=135°.
d. Draw tangent at A and B. then angle APB= 45°.

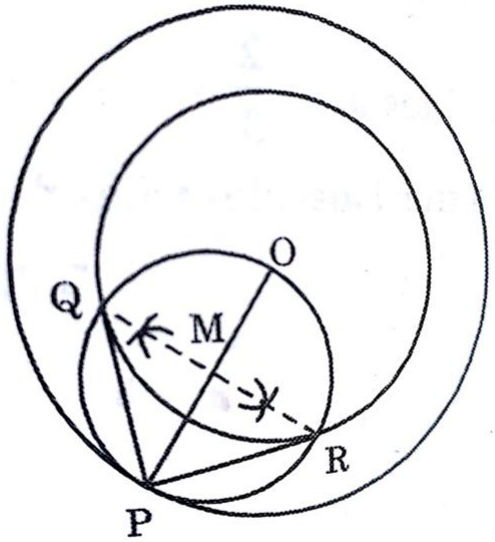
2



Steps of Constructions

a. Draw AB=6 cm.
b. With A and B as centres taking 6.6 cm and 7.5 CM as radii, draw two arcs intersecting each other at C.
c. Join triangle ABC as a given triangle.
d. Draw angle BAX an acute angle.
e. Along AX draw points A_1, A_2, A_3, A_4 at equal distances.
f. Join BA_4.
g. Draw $A_3B´$ parallel A4B which intersects AB at B´.
h. Draw B´C´ parallel BC which intersects AC at C´.
i. Hence triangle AB'C' is the required Triangle.

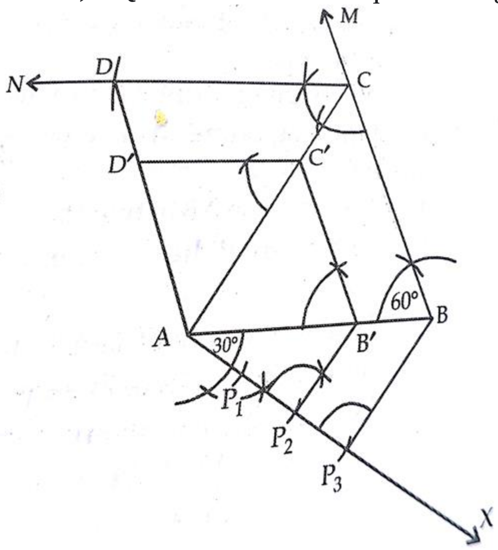
3



Steps of Constructions

a. Draw a circle of 3 cm radius with Centre O on the given plane.
b. Draw a circle of 5 cm radius radius taking O its Centre. locate a point P on the circle and join OP.
c. let M be the midpoint of PO.
d. Taking M as its centre its centre and MOas its radius ,draw a circle . let it intersect the given circle at points at point Q and R.
e. Join PQ and PR, PQ and PR are the required tangents.

4

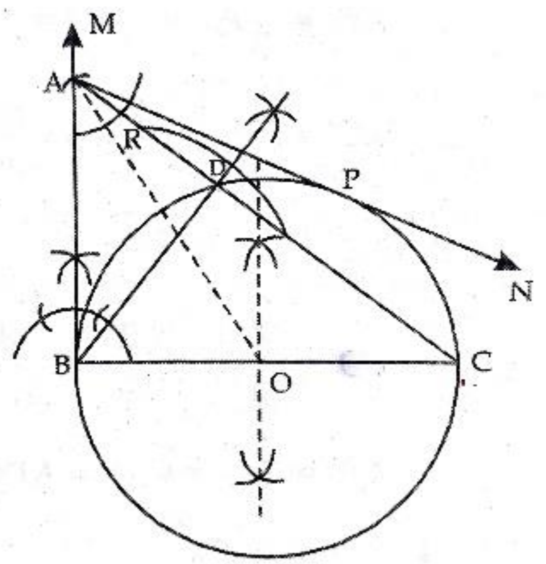


Steps of Constructions

a. Draw AB= 4 cm.
b. At B, draw an angle ABM =60° and cut off BC= 4 cm on BM.
c. At C draw an angle BCN= 120°.
d. Cut off CD = 4 cm on CN.
e. Join AD to complete the Rhombus ABCD.
f. Draw a Ray AX below the side AB such that angle BAX= 30°.
g. Mark 3 points P_1, P_2, P_3 on AX such that $AP_1= P_1 P_2= P_2 P_3$.
h. Join P_3B.
i. Draw a line $P_2B´$ parallel P_3B intersecting AB at B´.
j. Draw a line B´C´ parallel BC intersecting AC at C´.

   triangle AB´C´ is similar to triangle ABC with scale factor $\frac{2}{3}$.
k. Draw segment C´ D´ parallel CD.
l. ABCD is a rhombus.

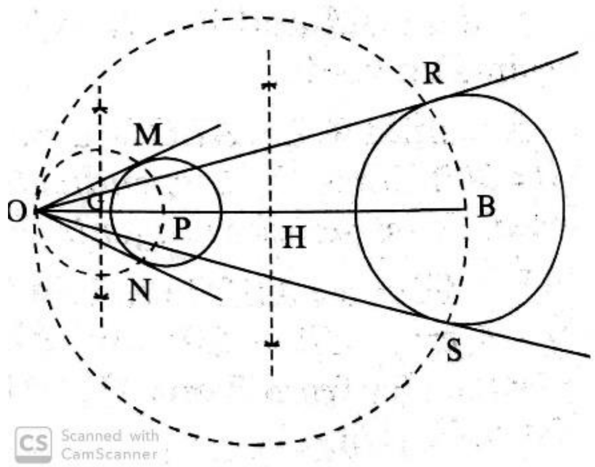
5



Steps of construction

a. Draw BC= 8 cm.
b. At B construct an angle CBM =90° and cut of BA =6 cm on BM.
c. Join AC to get given right angle triangle ABC.
d. Please compass at B and open its face so as to draw an Arc intersect AC at R and S.
e. Draw perpendicular bisectors of RS intersecting RS at D.
f. Join BD to get perpendicular from B on AC.
g. Draw perpendicular bisector of BC which meets BC at O.
   8. With O as centre and OB as radius draw a circle . Passing through B,C,D .
h. Join OA.
i. Draw a Ray AN through A such that angle BAO is equal to angle OAN .angle OBA=90°.
j. Thus AB and AP are required tangents to the circle

6



Steps of Constructions

a. Draw a line segment OB = 11 cm.
b. Mark a point P on OB, such that OP = 4 cm.
c. Draw a circle with centre P and radius = 1.5 cm.
d. Draw a circle with centre B and radius = 3 cm.
e. Draw the perpendicular bisector of OP. Let G be the mid point of OP.
f. With G as centre and radius = GP, draw a circle which intersects the circle with centre P at the two points M and N.

g. Join OM and ON. OM and ON are the two tangents from the point O to the circle with centre P.
h. By measurement OM = ON = 3.7cm.
i. Draw the perpendicular bisector of OB. Let H be the mid point of OB.
j. With H as centre and radius = HB, draw a circle which intersects the circle with centre B at two points R and S.
k. Join OR and OS. OR and OS are two tangents from the point O to the circle with centre B.
l. By measurement OR = OS = 10.4 cm.

CHAPTER-12 : AREAS RELATED TO CIRCLES

SECTION –A (One mark questions)

Multiple choice question(select the most appropriate answer from the following given options)

1.If the radius of a circle is 5.6cm then circumference of circle is

(a) 35.8cm (b) 70.7cm (c) 35.2cm (d) 70.5cm

2. If the diameter of a circle is 11.2cm then area of circle is

(a) 90.25cm^2 (b) 98.56cm^2 (c) 99.52cm^2 (d) 100cm^2

3.If the circumference of a circle is 123.2cm then radius of circle is

(a) 20cm (b) 20.6cm (c) 19cm (d)19.6cm

4. If the circumference of a circle is 123.2cm then area of circle,correct to nearest cm^2 is

(a) 1200cm^2 (b) 1205cm^2 (c)1207cm^2 (d)1210cm^2

5.The area of a circle is 301.84cm^2 then the radius of a circle is

(a)4.9cm (b)9.8cm (c) 10cm (d)10.5cm

6. The area of a circle is 301.84cm^2 then the circumference of a circle,correct to nearest cm is

(a) 60cm (b) 61cm (c)62cm (d)63cm

7 .The perimeter of a semicircular protector is 32.4cm then the radius of the protector is

(a)5.4cm (b) 5.7cm (c) 6.3cm (d)7.3cm

8. The perimeter of a semicircular protector is 32.4cm then the area of the protector is

(a)62.37cm^2 (b) 62.68cm^2 (c) 72.56cm^2 (d)77.7cm^2

SECTION –B (Two marks questions)

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Sol. We have, r_1 = 19 cm

r_2 = 9 cm

∴ Circumference of circle-I = $2\pi r_1 = 2\pi (19)$ cm

Circumference of circle-II = $2\pi r_1 = 2\pi (19)$ cm

Sum of the circumferences of circle-I and circle-II

$= 2\pi (19) + 2\pi (9) = 2\pi (19 + 9)$ cm $= 2\pi (28)$ cm

Let R be the radius of the circle-III.

∴ Circumference of circle-III = $2\pi R$

According to the condition,

$2\pi R = 2\pi (28)$

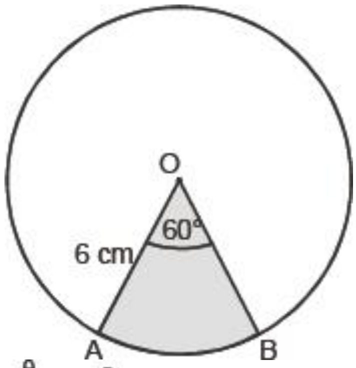
$\Rightarrow R = \frac{2\pi(28)}{2\pi} = 28$ cm

Thus, the radius of the new circle = 28 cm.

2. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Sol. Here, r = 6 cm

$\theta = 60°$



$\therefore$ Using, the Area of a sector $= \frac{\theta}{360} \times \pi r^2$

We have,
Area of the sector with r = 6 cm and $\theta = 60°$

$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6 \text{ cm}^2 = \frac{22}{7} \times 6 \text{ cm}^2 = \frac{132}{7} \text{cm}^2.$

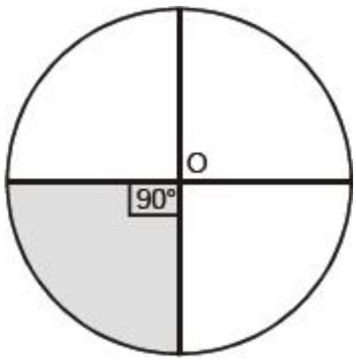
3. Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Let radius of the circle = r

$\therefore 2\pi r = 22$

$\Rightarrow 2 \times \frac{22}{7} r = 22$

$\Rightarrow r = 2 \times \frac{22}{7} \times \frac{1}{2} = \frac{7}{2} \text{cm}$



$\therefore$ Area of the quadrant $\left(\frac{1}{4}\text{th}\right)$ of the circle,

$= \frac{\theta}{360} \times \pi r^2 = \frac{90}{360°} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{cm}^2 = \frac{1 \times 11 \times 7}{4 \times 2} \text{cm}^2 = \frac{77}{8} \text{cm}^2.$

4. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Sol. [Length of minute hand] = [radius of the circle]

$\Rightarrow r = 14$ cm

θ Angle swept by the minute hand in 60 minutes = 360°

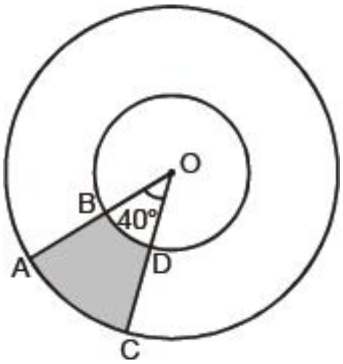
$\therefore$ Angle swept by the minute hand in 5 minutes $= \frac{360°}{60°} \times 5 = 30°$

Now, area of the sector with r = 14 cm and $\theta = 30°$

$\frac{\theta}{360} \times \pi r^2 = \frac{30}{360} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2 = \frac{11 \times 14}{3} \text{cm}^2 = \frac{154}{3} \text{cm}^2$

Thus, the required area swept by the minute hand by 5 minutes $= \frac{154}{3} \text{cm}^2.$

5. Find the area of the shaded region in figures, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40°$.



Sol. Radius of the outer circle = 14 cm

Here, θ = 40°

∴ Area of the sector BOD

$$= \frac{40}{360} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \frac{1}{9} \times 22 \times 7 \text{ cm}^2 = \frac{154}{9} \text{ cm}^2$$

Now, area of the shaded region

= [Area of sector AOC] – [Area of sector BOD]

$$= \frac{616}{9} - \frac{154}{9} \text{cm}^2 = \frac{1}{9}[616 - 154]\text{cm}^2 = \frac{1}{9} \times 462 \text{cm}^2$$

$$= \frac{1}{3} \times 154 \text{cm}^2.$$

SECTION –C (Three marks questions)

1. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Sol. Diameter of a wheel = 80 cm

$\therefore$ Radius of the wheel = $\frac{80}{2} = 40$ cm

$\therefore$ Circumference of the wheel

$$= 2\pi \times 40 = 2 \times \frac{22}{7} \times 40 \text{ cm}$$

$\Rightarrow$ Distance covered by a wheel in one revolution = $\frac{2 \times 22 \times 40}{7}$ cm

Distance travelled by the car in 1hr

= 66 km = 66 × 1000 × 100 cm

∴ Distance travelled in 10 minutes

$$= \frac{66 \times 1000 \times 100}{60} \times 10 \text{ cm} = 11 \times 100000 \text{ cm}$$

Now,

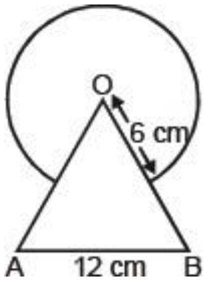
Number of revolutions

$$= \frac{[\text{Distance travelled in 10 minutes}]}{[\text{Distance travelled in one revolution}]}$$

$$= \frac{[1100000]}{\left[\frac{2 \times 22 \times 40}{7}\right]} = \frac{1100000 \times 7}{2 \times 22 \times 40} = 4375$$

Thus, the required number of revolutions = 4375.

2. Find the area of the shaded region in figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

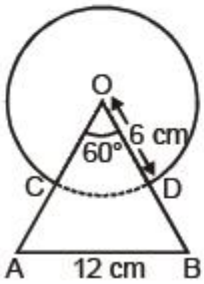


Sol. Area of the circle with radius = 6 cm.

$$= \pi r^2 = \frac{22}{7} \times 6 \times 6 \text{ cm}^2 = \frac{792}{7} \text{ cm}^2$$

Area of equilateral triangle, having side a = 12 cm, is given by

$$\frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 12 \times 12 \text{ cm}^2 = 36\sqrt{3} \text{ cm}^2$$



$\because$ Each angle of an equilateral triangle = 60°

$\therefore$ $\angle AOB = 60°$

$$\therefore \text{ Area of sector COD} = \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 6 \times 6 \text{ cm}^2$$

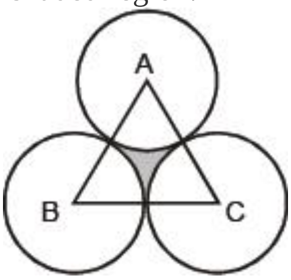
$$= \frac{22 \times 6}{7} \text{cm}^2 = \frac{132}{7} \text{cm}^2$$

Now, area of the shaded region,

= [Area of the circle] + [Area of the equilateral triangle] – [Area of the sector COD]

$$= \frac{792}{7} + 36\sqrt{3} - \frac{132}{7} \text{cm}^2 = \left[\frac{660}{7} + 36\sqrt{3}\right] \text{cm}^2.$$

3. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig.). Find the area of the shaded region.



Sol. Area of ΔABC = 17320.5 cm^2

$\because \Delta ABC$ is an equilateral triangle and area of an equilateral $\Delta = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$\therefore \frac{\sqrt{3}}{4}(\text{side})^2 = 17320.5$

$\Rightarrow \frac{173205}{4}(\text{side})^2 = 17320.5$

$\Rightarrow \frac{173205}{400000}(\text{side})^2 = \frac{173205}{10}$

$\Rightarrow (\text{side})^2 = \frac{173205}{10} \times \frac{400000}{173205}$

$\Rightarrow (\text{side})^2 = 40000$

$\Rightarrow (\text{side})^2 = (200)^2 \Rightarrow \text{side} = 200 \text{ cm}$

$\Rightarrow$ Radius of each circle $= \frac{200}{2} = 100$ cm

Since each angle of an equilateral triangle is 60°,

$\therefore \angle A = \angle B = \angle C = 60°$

$\therefore$ Area of a sector having angle of sector as 60° and radius 100 cm.

$= \frac{60}{360} \times \frac{314}{100} \times 100 \times 100 \text{ cm}^2 = \frac{1}{3} \times \frac{314}{100} \times 100 \times 100 \text{ cm}^2 = \frac{15700}{3} \text{cm}^2$

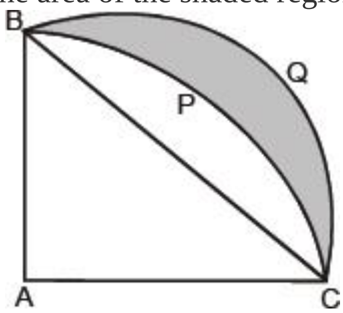
$\therefore$ Area of 3 equal sectors $= 3 \times \frac{15700}{3} \text{cm}^2 = 15700 \text{cm}^2$

Now, area of the shaded regino

= [Area of the equilateral triangle ABC] – [Area of 3 equal sectors]

= 17320.5 cm^2 – 15700 cm^2 = 1620.5 cm^2.

4. In the figure, ABPC is a quadrant of a circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of the shaded region.



Sol. Radius of the quadrant = 14 cm

Therefore, area of the quadrant ABPC

$= \left[\frac{90}{360} \times \frac{22}{7} \times 14 \times 4\right] \text{cm}^2 \left[\text{using} \frac{\theta}{360} \times \pi r^2\right]$

$= 22 \times 7 \text{ cm}^2 = 154 \text{ cm}^2$

Area of right $\Delta ABC = \frac{1}{2} \times 14 \times 14 \text{ cm}^2 = 98 \text{ cm}^2$

$\Rightarrow$ Area of segment BPC = 154 cm^2 – 98 cm^2 = 56 cm^2

Now, in right ΔABC,

$AC^2 + AB^2 = BC^2$

$\Rightarrow 142 + 142 = BC^2$

$\Rightarrow 196 + 196 = BC^2$

$\Rightarrow BC^2 = 392 \Rightarrow BC = 14\sqrt{2}$ cm.

$\therefore$ Radius of the semi-circle BQC $= \frac{14\sqrt{2}}{2}$ cm $= 7\sqrt{2}$ cm

$\therefore$ Area of the semi-circle BQC $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \left(7\sqrt{2}\right)^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} = 11 \times \sqrt{2} \times 7 \times \sqrt{2}$ cm^2

$= 11 \times 7 \times 2$ cm^2 $= 154$ cm^2

Now, area of the shaded region

= [Area of segment BQC] – [Area of segment BPC]

= 154 cm^2 – 56 cm^2 = 98 cm^2.

Side of the square = 8 cm

∴ Area of the square (ABCD) = 8 × 8 cm^2

= 64 cm^2

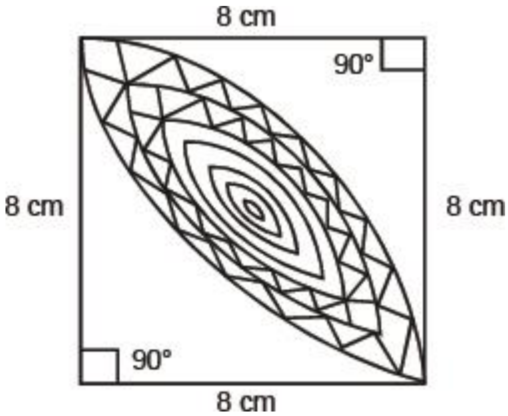
Now, radius of the quadrant ADQB = 8 cm

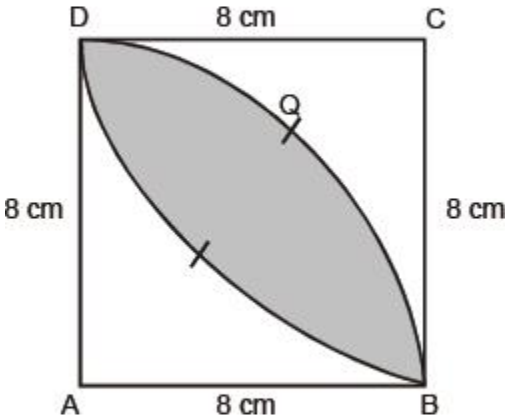
$\therefore$ Area of the quadrant ADQB $= \frac{90}{360} \times \frac{22}{7} \times 8^2$ cm^2

$= \frac{1}{4} \times \frac{22}{7} \times 64$ cm^2

$= \frac{22 \times 16}{7}$ cm^2

5. Calculate the area of the designed region in the figure, common between the two quadrants of circles of radius 8 cm each.





Similarly, area of the quadrant BPDC $= \frac{22 \times 16}{7} cm^2$

$\therefore$ Sum of the two quadrants $= 2\left[\frac{22 \times 16}{7}\right] cm^2 = \frac{704}{7} cm^2$

Now, area of design

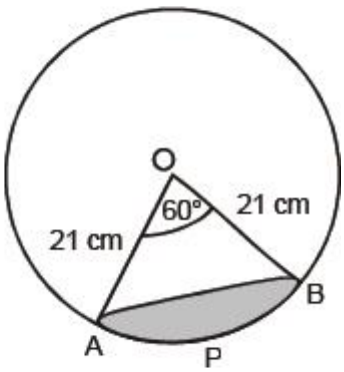
= [Sum of the areas of the two quadrants] – [Area of the square ABCD]

$$= \frac{704}{7} cm^2 - 64 cm^2 = \frac{704 - 448}{7} cm^2 = \frac{256}{7} cm^2.$$

SECTION –D (Four marks questions)

1. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
   (i) the length of the arc
   (ii) area of the sector formed by the arc
   (iii) area of the segment formed by the corresponding chord

Sol. Here, radius = 21 cm and θ = 60°



(i) Circumference of the circle = 2 πr

$$= 2 \times \frac{22}{7} \times 21 \text{ cm} = 2 \times 22 \times 3 \text{ cm} = 132 \text{ cm}$$

$$\therefore \text{Length of } \widehat{APB} = \frac{60}{360} \times 132 \text{ cm}$$

$$= \frac{1}{6} \times 132 \text{ cm} = 22 \text{ cm}$$

(ii) Area of the sector with sector angle 60°

$$= \frac{60°}{360°} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 11 \times 21 \text{ cm}^2 = 231 \text{ cm}^2$$

(iii) Area of the segment APQ

= [Area of the sector AOB] – [Area of ΔAOB] ...(1)

In ΔAOB, OA = OB = 21 cm

∴∠A = ∠B = 60° [θ∠O = 60°]

⇒AOB is an equilateral Δ,
∴AB = 21 cm
Draw OM ⊥ AB such that

$$\frac{OM}{OA} = \sin 60° = \frac{\sqrt{3}}{2} \Rightarrow OM = 21 \times \frac{\sqrt{3}}{2} \text{cm}$$

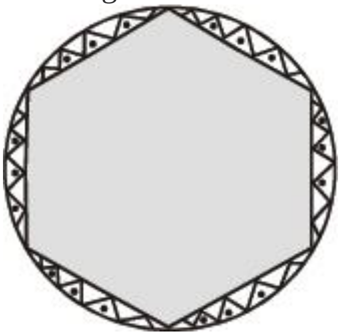
$$\text{Now area of a } \Delta OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21 \times 21 \times \frac{\sqrt{3}}{2} \text{ cm}^2$$

$$= \frac{441\sqrt{3}}{4} \text{cm}^2$$

From (1) and (2), we have:

$$\text{Area of segment} = [231 \text{ cm}^2] - \left[\frac{441\sqrt{3}}{4} \text{cm}^2\right] = \left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2.$$

2. A round table cover has six equal designs as shown in Fig. If the radius of the cover is 28 cm, find the cost of makign the designs at the rate of Rs. 0.35 per cm^2.



Sol. Here, r = 28 cm

Since, the circle is divided into six equal sectors.

$$\therefore \text{ Sector angle } \theta = \frac{360°}{6} = 60°.$$

∴ Area of the sector with θ = 60° and r = 28 cm

$$= \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 \text{ cm}^2$$

$$= \frac{44 \times 28}{3} \text{cm}^2 = 410.67 \text{ cm}^2 \qquad ...(1)$$

Now, area of 1 design
= Area of segment APB
= Area of sector – Area of
ΔAOB ...(2)

In ΔAOB, ∠AOB = 60°, OA = PN = 28 cm
∴ ∠OAB = 60° and ∠OBA = 60°
⇒ ΔAOB is an equilateral triangle.
⇒ AB = AO = BO
⇒ AB = 28 cm
Draw OM ⊥AB
∴ In right ΔAOM, we have

$$\frac{OM}{OA} = \sin 60° = \frac{\sqrt{3}}{2} \Rightarrow OM = OA \times \frac{\sqrt{3}}{2} cm$$

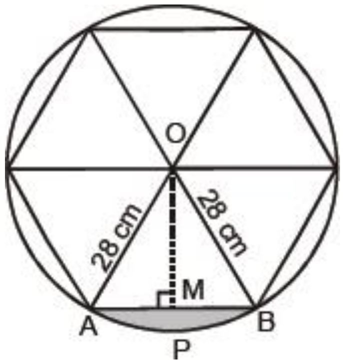
$$\Rightarrow OM = 28 \times \frac{\sqrt{3}}{2} cm$$

$$\Rightarrow OM = 14\sqrt{3}\ cm$$

$$\therefore \text{ Area of } \Delta AOB = \frac{1}{2} AB \times OM = \frac{1}{2} \times 28 \times 14\sqrt{3}\ cm^2$$

$$= 14 \times 14\sqrt{3}\ cm^2$$

$$= 14 \times 14 \times 1.7\ cm^2 = 333.3\ cm^2$$



Now, from (1), (2) and (3), we have:

Area of segment APQ = 410.67 cm^2 – 333.2 cm^2 = 77.47 cm^2

⇒ Area of 1 design = 77.47 cm^2

∴ Area of the 6 equal designs = 6 × (77.47) cm^2

= 464.82 cm^2

Cost of making the design at the rate of Rs. 0.35 per cm^2,

= Rs. 0.35 × 464.82

= Rs. 162.68.

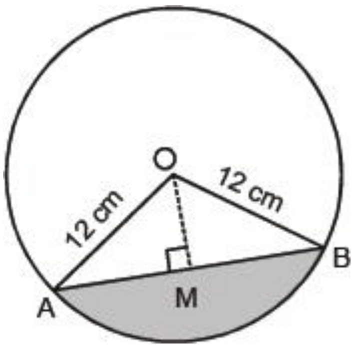
3. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

Sol. Here, θ = 120° and r = 12 cm

$$\therefore \text{ Area of the sector} = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{120}{360} \times \frac{314}{100} \times 12 \times 12\ cm^2$$

$$= \frac{314 \times 4 \times 12}{100} cm^2 = \frac{15072}{100} cm^2 = 150.72\ cm^2 \qquad ...(1)$$



Now, area of $\Delta AOB = \frac{1}{2} \times AB \times OM$ ...(2)

In ΔOAB, $\angle O = 120°$

$\Rightarrow \angle A + \angle B = 180° - 120 = 60°$

$\because$ $OB = OA = 12$ cm $\Rightarrow \angle A = \angle B = 30°$

So, $\frac{OM}{OA} = \sin 30° = \frac{1}{2} \Rightarrow OM = OA \times \frac{1}{2}$

$\Rightarrow OM = 12 \times \frac{1}{2} = 6$ cm

In right ΔAMO, $12^2 - 6^2 = AM^2$

$\Rightarrow 144 - 36 = AM^2$

$\Rightarrow 108 = AM^2$

$\Rightarrow$ $AM = \sqrt{108} = 6\sqrt{3}$

$\Rightarrow$ $2\,AM = 12\sqrt{3}$

$\Rightarrow$ $AB = 12\sqrt{3}$

Now, from (2),

Area of $\Delta AOB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 12\sqrt{3} \times 6\ cm^2 = 36\sqrt{3}\ cm^2$

$= 36 \times 1.73\ cm^2 = 62.28\ cm^2$ ...(3)

From (1) and (3)

Area of the minor segment

= [Area of minor segment] – [Area of ΔAOB]

= [150.72 cm^2] – [62.28 cm^2] = 88.44 cm^2.

4. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

Sol. Here, radius (r) = 15 cm

Sector angle $\theta = 60°$

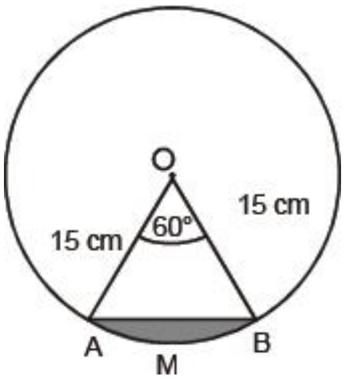
$\therefore$ Area of the sector with $\theta = 60°$

$$= \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \frac{314}{100} \times 15 \times 15\ cm^2 = \frac{11775}{100} cm^2 = 117.75\ cm^2$$

Since $\angle O = 60°$ and OA = OB = 15 cm

$\angle AOB$ is an equilateral triangle.

$\Rightarrow$ AB = 15 cm and $\angle A = 60°$



Draw OM ⊥ AB

$\therefore \frac{OM}{OA} = \sin 60° = \frac{\sqrt{3}}{2}$

$\Rightarrow OM = OA \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{2}$ cm

Now, ar(Δ AOB) = $\frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 15 \times 15\frac{\sqrt{3}}{2} \text{ cm}^2 = \frac{225\sqrt{3}}{4} \text{cm}^2$

Now area of the minor segment

= (Area of minor sector) – (ar ΔAOB)

= (117.75 – 97.3125) cm^2 = 20.4375 cm^2

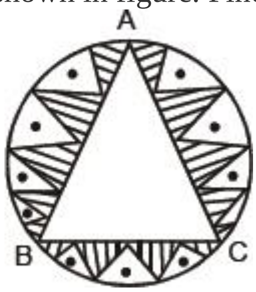
Area of the major segment

= [Area of the circle] – [Area of the minor segment]

$= \pi r^2 - 20.4375 \text{ cm}^2 = \left[\frac{314}{100} \times 15^2\right] - 20.4375 \text{ cm}^2$

= 706.5 – 20.4375 cm^2 = 686.0625 cm^2.

5. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design.



Sol. Area of the circle having radius r = 32 cm.

$= \pi r^2$

$= \frac{22}{7} \times 32 \times 32 \text{ cm}^2 = \frac{22528}{7} \text{cm}^2$

‘O’ is the centre of the circle,

∴ AO = OB = OC = 32 cm

⇒ ∠AOB = ∠BOC = ∠AOC = 120°

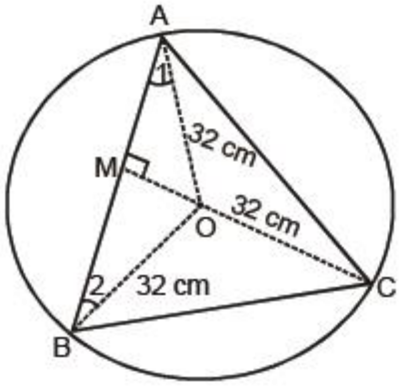
Now, in ΔAOB, ∠1 = 30°

∵ ∠1 + ∠2 = 60°

Also OA = OB ⇒ ∠1 = ∠2

If OM ⊥AB, then

$\frac{OM}{OA} = \sin 30° = \frac{1}{2} \Rightarrow OM = OA \times \frac{1}{2}$



$\Rightarrow\ OM = 32 \times \frac{1}{2} = 16$ cm

Also, $\frac{AM}{AO} = \cos 30° = \frac{\sqrt{3}}{2}$

$\Rightarrow AM = \frac{\sqrt{3}}{2} \times AO = \frac{\sqrt{3}}{2} \times 32$

$\Rightarrow 2AM = AB = 2 \times \left(\frac{\sqrt{3}}{2} \times 32\right) = 32\sqrt{3}$ cm

Now, area of ΔAOB, $= \frac{1}{2} \times OM \times AB = \frac{1}{2} \times 16 \times 32 \times \sqrt{3} = 256\sqrt{3}$ cm^2

Since area $\Delta ABC = 3 \times$ [area of ΔAOB] $= 3 \times 256 \times \sqrt{3}$ cm$^2 = 768\sqrt{3}$ cm^2

Now, area of the design = [Area of the circle] – [Area of the equilateral triangle]

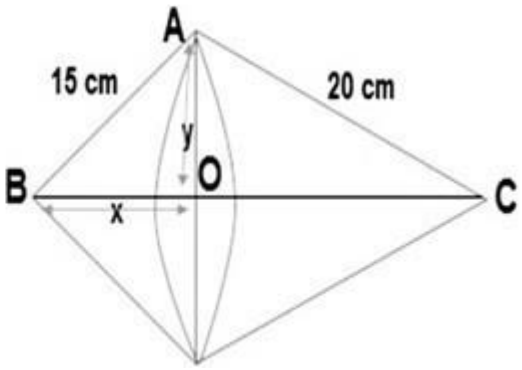
$= \left(\frac{22528}{7} - 768\sqrt{3}\right)$cm^2.

CHAPTER- 13 : SURFACE AREAS AND VOLUMES
(VOLUME OF COMBINATION OF SOLIDS AND CONVERSION OF SOLIDS)

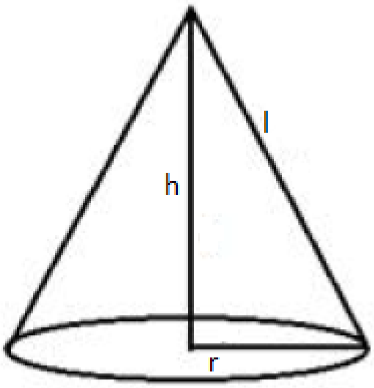
1	A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylindrical tub is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub (use $\pi = \frac{22}{7}$]
2	A right triangle whose sides are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume of the double cone so formed. (use = 3.14)
3	Water in a canal 30 dm wide and 12 dm deep is flowing with a velocity of 10 km/h. How much area will it irrigate in 30 minutes if 8 cm of standing water is required for irrigation?
4	A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16 cm and height 42 cm. The total space between two vessels is filled with cork dust for heat insulation purposes. How many cubic centimetres of cork dust will be required?
5	A building is in the form of a cylinder surrounded by a hemispherical vaulted dome and contains $41\frac{19}{21}$ - cu m of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building.
6	Two right circular cones X and Y are made X having 3 times the radius of Y and Y having half the Volume of X. Calculate the ratio of heights of X and Y
7	A vessel in shape of a inverted cone is surmounted by a cylinder has a common radius of 7cm this was filled with liquid till it covered one third the height of the cylinder. If the height of each part is 9cm and the vessel is turned upside down. Find the volume of the liquid and to what height will it reach in the cylindrical part.
8	Circumference of the edge of a hemispherical bowl is 132cm. find the capacity of the bowl.
9	Marbles of diameter 1.4cm are dropped into a cylindrical beaker containing some water and are fully submerged. The diameter of the beaker is 7cm.Find how many marbles have been dropped in it if water rises by 56cm.
10	A rectangular piece of paper is 22 cm long and 10 cm wide . A cylinder is formed by rolling in such a way that the length of the paper becomes height of the cylinder. Find the volume of the cylinder
11	Three cubes of a metal whose edges are in the ratio 3:4:5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.
12	A pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold the pens and pins, respectively. The dimension of the cuboid are 10 cm, 5 cm and 4 cm. The radius of each of the conical depressions is 0.5 cm and the depth is 2.1 cm. The edge of the cubical depression is 3 cm. Find the volume of the wood in the entire stand.

13	The internal and external radii of a hollow spherical shell are 3 cm and 5cm respectively. if it is melted to from a solid cylinder of height 2⅔cm ,find the diameter of the cylinder
14	A well with 14 m inside diameter is dug 8 m deep. The earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. Find the height of the embankment.
15	A mason constructs a wall of dimensions 270cm× 300cm × 350cm with the bricks each of size 22.5cm × 11.25cm × 8.75cm and it is assumed that 1/8 space is covered by the mortar. Then find the number of bricks used to construct the wall

Solutions

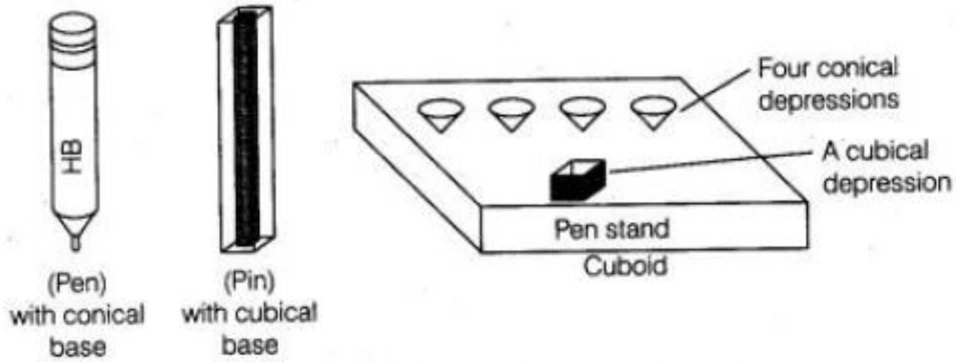
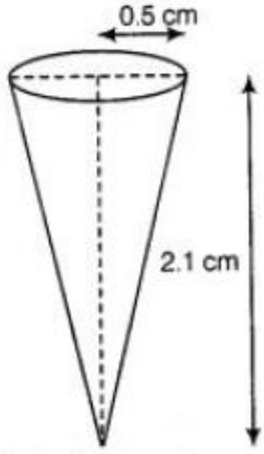
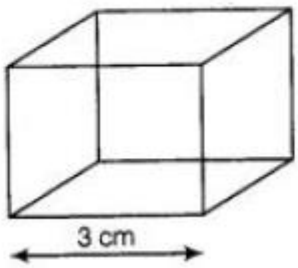
1	Given, r=3.5cm height of conical part =4cm For cylindrical tub R=5cm H=10.5cm Volume of water left in the cylindrical tub = Volume of cylinder-(volume of cone + volume of hemisphere) $=\pi R^2H-((\pi/3)r^2h+2/3\pi r^3)$ $=(825-(51.33+89.83))cm^3$ (put (π=22/7) $=683.84cm^3$
2	 When a right-angled triangle is revolved around its hypotenuse, a double cone is formed with same radius but with different heights. it is given that, AB = 15 cm, AC = 20 cm Let, OB = x and OA = y Observe from the figure, In right-angled triangle ABC, By Pythagoras theorem [Hypotenuse^2 = Base^2 + Perpendicular^2] $BC^2 = AC^2 + AB^2$ $\Rightarrow BC^2 = 20^2 + 15^2$ $\Rightarrow BC^2 = 400 + 225$ $\Rightarrow BC^2 = 625$ $\Rightarrow BC = 25$ cm In ΔOAB

	$AB^2 = OA^2 + OB^2$ $\Rightarrow 15^2 = x^2 + y^2$ ……(1) In ΔAOC $AC^2 = OA^2 + OC^2$ $\Rightarrow 20^2 = y^2 + (BC - OB)^2$ $\Rightarrow 400 = y^2 + (25 - x)^2$ $\Rightarrow 400 = y^2 + 625 - 50x + x^2$ $\Rightarrow 400 = 15^2 + 625 - 50x$ $\Rightarrow 400 = 225 + 625 - 50x$ $\Rightarrow 50x = 450$ $\Rightarrow x = 9$ cm from equation (1), $15^2 = 9^2 + y^2$ $\Rightarrow y^2 = 225 - 81$ $\Rightarrow y^2 = 144$ $\Rightarrow y = 12$ cm Also, OC = 25 – x = 25 – 9 = 16 cm^2 Now, Volume of cone , V = 1/3 $\pi r^2 h$ Hence, volume of double cone $= 1/3\ \pi(OA)^2 \times BO + 1/3\ \pi(OA)^2 \times OC$ $= 1/3\ \pi(12)^2 \times (OB + OC)$ $= 1/3 \times 3.14 \times 144 \times 25$ $= 3768\ cm^3$
3	Width of the canal = 30 dm = 300 cm = 3 m Depth of the canal 12 cm = 120 cm = 1.2 m It is given that the water is flowing with velocity 10 km/hr Therefore Length of the water column formed in 1/2 hour = 5 km = 5000 m Therefore volume of the water flowing in 1 hour = Volume of the cuboid of length 5000 m, width 3 m and depth 1.2 m => Volume of the water in half hour = (5000*3*1.2) m3 = 18000 m^3 Suppose x m2 area is irrigated in 1/2 hour as 8cm standing water is required Therefore x*8/100 = 18000 x = 1800000/8 x = 225000m^2 Hence the 225000 m^2 area irrigated in 1/2 hours
4	Given height of cylindrical vessel (h) = 42cm Inner radius of a vessel (r1) = 14/2 cm =7 cm Inner radius of a vessel (r2) = 16/2 cm = 8cm Volume of a cylinder $= \pi\left(r_2^2 - r_1^2\right)h$ $= \pi\left(8^2 - 7^2\right)42$ $= \pi\left(64 - 49\right)42$ $= 15 \times 42 \times \pi$ $= 630\pi$ $= 1980cm^3$ Volume of a vessel $= 1980cm^2$

5	Let r be the radius of hemisphere & Cylinder and h be the height of the Cylinder, H be the height of the Total building Volume of air = $880/21\ m^3$ Internal diameter (d) = H Internal Diameter = 2r = H Total Height of the building (H) = 2r……(1) Height of the building = height of the cylinder + radius of the hemispherical Dome $H = h + r$ $2r = h + r$ [from eq 1] $2r - r = h$ $r = h$ ……………..(2) Volume of air inside the building = Volume of cylindrical portion + Volume of hemispherical portion $\pi r^2 h + (2\pi r^3/3) = 880/21$ $\pi(h)^2 h + (2\pi(h)^3/3) = 880/21$ [From eq 2, r= h] $\pi h^3 + \frac{2}{3}\pi h^3 = 880/21$ $\pi h^3(1+\frac{2}{3}) = 880/21$ $\pi h^3[5/3] = 880/21$ $22/7 \times h^3 \times 5/3 = 880/21$ $h^3 = (880 \times 3 \times 7) / 21 \times 22 \times 5$ $h^3 = 40/5 = 8$ $h^3 = 8$ $h = \sqrt[3]{8} = \sqrt[3]{2\times2\times2}$ $h = 2$ m $h = r = 2$ m [From eq 2, r= h] Total height of the building(H) = 2r = 2×2 = 4 m Hence, the Total height of the building is 4m.
6.	Let radius of cone y = r Therefore, radius of cone x = 3r Let volume of cone y = V then volume of cone x = 2V Let h1 be the height of x and h2 be the height of y. 

	Therefore, Volume of cone $= \frac{1}{3}\pi r^2 h$ Volume of cone x $= \frac{1}{3}\pi(3r)^2 h_1$ $= \frac{1}{3}\pi 9r^2 h_1 = 3\pi r^2 h_1$ Volume of cone y $= \frac{1}{3}\pi r^2 h_2$ $\therefore \frac{2V}{v} = \frac{3\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2}$ $\Rightarrow \frac{2}{1} = \frac{3h_1 \times 3}{h_2} = \frac{9h_1}{h_2}$ $\Rightarrow \frac{h_1}{h_2} = \frac{2}{1} \times \frac{1}{9} = \frac{2}{9}$ $\therefore h_1 : h_2 = 2 : 9$
7	Volume of liquid in the vessel = $\frac{1}{3}\pi$ $(7)^2$ (9) + π $(7)^2$ (3) = 924 cu cm height of cylindrical part = $\frac{924}{\frac{22}{7} \times 49}$ = 6 cm
8.	First we need to know about radius ..so, circumference=132 cm $2\pi r$ = 132 cm 2 × 22/7× r = 132 44/7 x r = 132 r= 132 x 7/44 r = 21 volume of hemisphere = 2/3 πr^3 =2/3 x 22/7 x 21x21x21 =44x 441 =19404 cm^3
9	Diameter of marble = 1.4 cm radius = 1.4/2 = 0.7 cm volume of 1 marble = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times (0.7)^3$

	let number of marbles required = n base diameter of beaker = 7cm radius = 7/2 = 3.5 cm height rise in water = 56 cm volume change = volume of n marbles $\pi r^2 h = n \times 4/3\ \pi(0.7)^3$ $n = \frac{3 \times 3.5 \times 3.5 \times 56}{4 \times 0.7 \times 0.7 \times 0.7}$ $= 1500$ So 1500 marbles should be dropped.
10	if we roll this rectangular piece then height of cylinder so formed will be 22 cm and the circumference of base of cylinder will be 10cm so, $2\pi r=10$ $2\times22/7\times r=10$ $r=10\times7/44$ $r=5\times7/22$ volume of cylinder $=\pi\times r\times r\times h$ $=22/7\times35/22\times35/22\times22$ $=5\times35 cm^3$ $=175 cm^3$
11	Let the edges of three cubes be 3x, 4x and 5x respectively. Volume of three cubes $= (3x)^3 + (4x)^3 + (5x)^3 = 216x^3\ cm^3$ Let a be the edge of new cube so formed. Now, Volume of new cube = Volume of three cubes $\Rightarrow a^3 = 216x^3$ $\Rightarrow a = 6x$ Diagonal of new cube $= 12\sqrt{3}$ cm $\Rightarrow \sqrt{a^2+a^2+a^2} = 12\sqrt{3}$ $\Rightarrow \sqrt{3}a = 12\sqrt{3}$ $\Rightarrow a = 12$ $\Rightarrow x = 2$ $\Rightarrow 3x = 6$ cm, $4x = 8$cm and $5x = 10$ cm

12	Given that, length of cuboid pen stand (l) = 10 cm Breadth of cubiod pen stand (b) = 5 cm and height of cuboid pen stand (h) = 4 cm  $\therefore$ Volume of cuboid pen stand $= l \times b \times h = 10 \times 5 \times 4 = 200\ \text{cm}^3$ Also, radius of conical depression $(r) = 0.5$ cm and height (depth) of a conical depression $(h_1) = 2.1$ cm $\therefore$ Volume of a conical depression $= \frac{1}{3}\pi r^2 h_1$ $= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2.1$ $= \frac{22 \times 5 \times 5}{1000} = \frac{22}{40} = \frac{11}{20} = 0.55\ \text{cm}^3$ Also, given Edge of cubical depression $(a) = 3$ cm $\therefore$ Volume of cubical depression $= (a)^3 = (3)^3 = 27\ \text{cm}^3$ So, volume of 4 conical depressions $= 4 \times$ Volume of a conical depression $= 4 \times \frac{11}{20} = \frac{11}{5}\ \text{cm}^3$ Hence, the volume of wood in the entire pen stand = Volume of cuboid pen stand − Volume of 4 conical depressions − volume of a cubical depressions $= 200 - \frac{11}{5} - 27 = 200 - \frac{146}{5}$ $= 200 - 29.2 = 170.8\ \text{cm}^3$  Conical depression  Cubical depression
13	Volume of the iron used to form spherical shell = $4/3\ \pi\ (R^3 - r^3)$ $= (4/3)\ (22/7)\ (5^3 - 3^3)$ $= (4/3)\ (22/7)\ (98)$ $= 410.67\ \text{cm}^3$ Height of the cylinder formed = 8/3 cm Let the radius of the cylinder be 'r' cm Volume of the cylinder = $\boldsymbol{\pi r^2 h}$ **Volume of the cylinder** = $22/7 \times r^2 \times 8/3$ $410.67 = 22/7 \times r^2 \times 8/3$ $r^2 = 49$ r = 7 cm $\therefore$ Diameter of the cylinder = 2 x 7 =14 cm
14	volume of earth dug out = $22/7*7*7*8 = 1232\ \text{cm}^3$ Area of embankment = Pi $(R^2 - r^2)$

	$= 22/7 * (28^2 - 7^2)$ $= 22/7 * 35 * 21$ $= 2310\ m^2$ Therefore Height of embankment 1232/2310 * 100 cm = 53.3 cm
15	The entire volume of the wall = 270cm× 300cm × 350cm $= 28350000\ cm^3$ If the mortar covers a space of 1/8 then the bricks covers a volume of 7/8 That is 7/8 of 28350000 = $24806250\ cm^3$ Now the volume of each brick= 22.5cm × 11.25cm × 8.75cm $= 2214.84375\ cm^3$ to find the number of bricks used divide the total volume by the volume of the bricks number of bricks = 24806250 ÷ 2214.84375 = 11200 bricks

PROBLEMS BASED ON CONVERSION OF SOLIDS

1. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylindrical tub is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub (use $\pi = \frac{22}{7}$] (Ans: $683.83\ cm^3$)

2. A bucket of height 8 cm and made up of copper sheet is in the form of frustum of right circular cone with radii of its lower and upper ends as 3 cm and 9 cm respectively. Calculate

   i) the height of the cone of which the bucket is a part

   ii) the volume of water which can be filled in the bucket

   iii) the area of copper sheet required to make the bucket (Leave the answer in terms of π

   (Ans: $129\ \pi\ cm^2$)

3. A sphere and a cube have equal surface areas. Show that the ratio of the volume of the sphere to that of the cube is $\sqrt{6} : \sqrt{\pi}$.

4. A right triangle whose sides are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed.

(Ans: 3768cu.cm,1318.8 Sq.cm)

5. Water in a canal 30 dm wide and 12 dm deep is flowing with a velocity of 10 km/h. How much area will it irrigate in 30 minutes if 8 cm of standing water is required for irrigation?

(Ans: 225000 cu. m)

6. A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16 cm and height 42 cm. The total space between two vessels is filled with cork dust for heat insulation purposes. How many cubic centimetre of cork dust will be required?

(Ans:1980 cu.cm)

7. An ice-cream cone has a hemispherical top. If the height of the cone is 9 cm and base radius is 2.5 cm, find the volume of ice cream cone. (Ans: $91\frac{2}{3}$ cu.cm)

8. A building is in the form of a cylinder surrounded by a hemispherical vaulted dome and contains $41\ \frac{19}{21}$ - cu m of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building. (Ans: 4m)

9. The height of the Cone is 30 cm A small cone is cut off at the top by a plane parallel to its base if its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section cut ? (Ans:20 cm)

10. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ th of the curved surface of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane. (Ans:1:2)

11. Two right circular cones X and Y are made X having 3 times the radius of Y and Y having half the Volume of X. Calculate the ratio of heights of X and Y. (Ans: 9: 2)

12. If the areas of three adjacent faces of cuboid are x, y, z respectively, Find the volume of the cuboids.

13. A shuttlecock used for playing badminton has the shape of a frustum of a Cone mounted on a hemisphere. The external diameters of the frustum are 5 cm and 2 cm, and the height of the entire shuttlecock is 7cm. Find the external surface area.

(Ans: 74.26cm^2)

14. A Solid toy in the form of a hemisphere surmounted by the right circular cone of height 2cm and diameter of the base 4 cm. If a right circular cylinder circumscribes the toy, find how much more space than the toy it will cover. (Ans: 8π)

15. A conical vessel of radius 6cm and height 8cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in the figure. What fraction of water flows out?

[Ans: $\frac{3}{8}$]

16. A golf ball has a diameter equal to 4.1cm. Its surface has 150 dimples each of radius 2mm. Calculate the total surface area which is exposed to the surroundings assuming that the dimples are
hemispherical.

(Ans: 71.68)

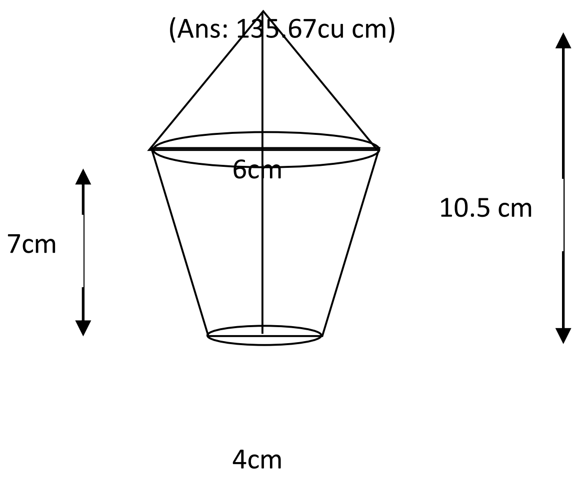
17. A solid metallic circular cone 20cm height with vertical angle 60 is cut into two parts at the middle point of its height by a plane parallel to the base. If the frustum, so obtained be drawn into a wire of diameter $\frac{1}{16}$cm Find the length of the wire.

(Ans:7964.4m)

18. If the areas of the circular bases of a frustum of a cone are $4cm^2$ and $9cm^2$ respectively and the height of the frustum is 12cm. What is the volume of the frustum? (Ans:$44cm^2$).

19. The lower portion of a hay stack is an inverted cone frustum and the upper part is a cone find the total volume of the hay stack.

(Ans: 135.67cu cm)



20. A vessel in shape of a inverted cone is surmounted by a cylinder has a common radius of 7 cm this was filled with liquid till it covered one third the height of the cylinder. If the height of each part is 9cm and the vessel is turned upside down. Find the volume of the liquid and to what height will it reach in the cylindrical part. Ans:924π cu cm, 6cm)

Answers

Ans: - 1 No. of solid = vol of cone + vol of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 [h + 2r]$$

On substituting we get,

$= 141.17\ cm^3$

vol of cylinder = $\pi r^2 h$

On substituting we get,

$= 825\ cm^3$

volume of H_2O left in the cylinder = 825 – 141.17

$= 683.83\ cm^3$

Ans: - 2 Let total height be h

$\Rightarrow \frac{h}{h+8} = \frac{3}{9}$ (similar Δ's)

=> h = 4 cm

$\therefore$ ht. of cone which bucket is a part = 4 cm

Substitute to get Ans.: for ii) iii)

Ans: - 3 S.A. of sphere = S.A of cube

⇨ $4\pi r^2 = 6a^2$

⇨ $r = \sqrt{\frac{6a^2}{4\Pi}}$

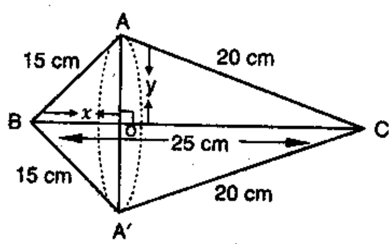
$\therefore$ ratio of their volume $\dfrac{v_1}{v_2} = \dfrac{\frac{4}{3}\Pi \gamma^3}{a^3}$

On simplifying & substituting, we get √6 : √π

Ans: - 4 BC = $\sqrt{15^2 + 20^2}$ = 25 cm

Apply Py. Th to right Δ OAB & OAC to get OB = 9cm OA = 12cm

Vol of double cone = vol of CAA^1 + vol of BAA^1



$= \frac{1}{3}\pi \times 12^2 \times 16 + \frac{1}{3}\pi \times 12^2 \times 9$

= 3768 cm^3

SA of double cone = CSA of CAA^1 + CSA of BAA^1

$= \pi \times 12 \times 20 + \pi \times 12 \times 15$

= 1318.8 cm^3

Ans: - 5 Width of canal = 30 dm = 3m

Depth of canal = 1.2 m

Velocity = 10 km / h = 10000 m/h

Length of water column is formed in 30 min = 10000 x $\frac{1}{2}$ = 5000 m

Let x m^2 of area be irrigated => $x \times \frac{8}{100} = 5000 \times 1.2 \times 3$

=> x = 225000 m^2

Ans: - 6 volume of cork dust required $= \pi R^2 h - \pi r^2 h$

$= \pi\, 42\, [64 - 49]$

$= 1980\ cm^3$

Ans: - 7 Do yourself

Ans: -8 Volume of building = $41\frac{19}{21}\ m^3$

$\Rightarrow \pi r^2 r + \frac{2}{3}\pi r^3 = 41\frac{19}{21}$

$\Rightarrow \pi \times r^3 \times \frac{5}{3} = \frac{880}{21}$

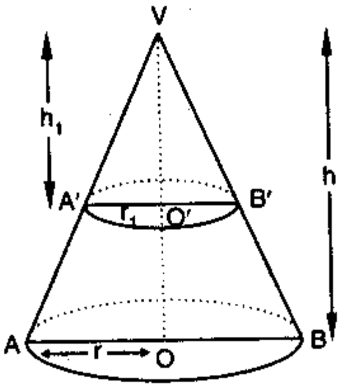
$\Rightarrow r^3 = \frac{880}{21} x \frac{7}{22} x \frac{3}{5}$

$\Rightarrow r^3 = 8$

$\Rightarrow r = 2$ m

$\therefore$ height of building = 4 cm

Ans: - 9 $\Delta\ VO^1B \sim \Delta VOB$



$\therefore \frac{H}{h} = \frac{R}{r} = \frac{30}{h} = \frac{R}{r}$ -----(1)

APQ: vol of cone $VA^1B^1 = \frac{1}{27}$ (vol of cone VAB)

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{1}{27}\left(\frac{1}{3}\pi R^2 H\right)$

$\Rightarrow h^3 = 1000$ (using (1)

h = 10 cm

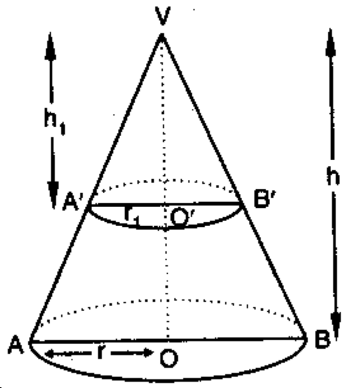
$\therefore$ height at which section is made (30 – 10) = 20 cm

Ans: - 10 We know that Δ VO[1]B $\sim \Delta$VOB

$$\frac{h}{H} = \frac{r}{R} = \frac{l}{L}$$

C. SA of frustum = $\frac{8}{9}$ (CSA of the cone)

Π (R +r) (L-l) = $\frac{8}{9}\Pi$ RL



$$\Rightarrow \left(\frac{R+r}{R}\right)\left(\frac{L-l}{L}\right) = \frac{8}{9}$$

$$\Rightarrow \left(1+\frac{r}{R}\right)\left(1-\frac{l}{L}\right) = \frac{8}{9}$$

$$\Rightarrow \left(1+\frac{h}{H}\right)\left(1-\frac{h}{H}\right) = \frac{8}{9}$$

On simplifying we get $\frac{h^2}{H^2} = \frac{1}{9}$

$$\frac{h}{H} = \frac{1}{3}$$

$\Rightarrow$ H = 3h

required ratios = $\frac{h}{H-h} = \frac{1}{2}$

Ans: - 11 Let radius of cone X = r

Radius of Cone Y = 3r

V of Y = $\frac{1}{2}$ volume of X

$\frac{1}{3}\pi r^2{}_1 h_1 = \frac{1}{2}\left(\frac{1}{3}\pi r^2{}_2 h_2\right)$

=> $r^2 h_1 = \frac{1}{2}\, 9 r^2 h_2$

$$\frac{h_1}{h_2} = \frac{9r^2}{2r^2}$$

$$\frac{h_1}{h_2} = \frac{9}{2}$$

Ans: - 12 $lb = x$, $bh = y$, $hl = z$

Volume of cuboid = l b h

$V^2 = l^2 b^2 h^2 = xyz$

$V = \sqrt{xyz}$

Ans: -13 r_1 = radius of lower end of frustum = 1 cm

r_2 = radius of upper end = 2.5 cm

h = ht of frustum = 6cm

$l = \sqrt{h^2 + (r_2 - r_1)^2}$ = 6.18 cm

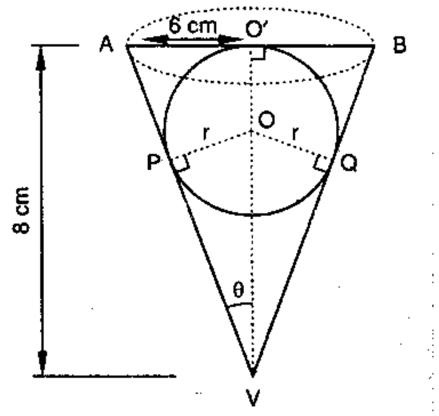
External surface area of shuttlecock = $\pi (r_1 + r_2) l + 2\pi r^2{}_1$

On substituting we get = 74.26 cm^2

Ans: - 14 Self practice

Ans: - 15 This problem can be done in many ways

Let "r" be the radius of sphere



In right triangle, tan $\theta = \frac{6}{8} = \frac{3}{4}$

=> Sin $\theta = \frac{3}{5}$

In rt Δ

Sin $\theta = \frac{r}{VO} = \frac{3}{5} = \frac{r}{8-r}$

r = 3 cm

Volume of H_2O that flows out of cone = volume of sphere

Fraction of water Overflows = $\frac{\text{volume f sphere}}{\text{Volume of cone}}$

$= \frac{36\,\pi}{96\,\pi} = \frac{3}{8}$

Ans: - 16 SA of ball = $4\pi \times (\frac{4.1}{2})^2 = 16.8\,\pi$ cm^2

TSA exposed to surroundings

= SA of ball – 150 $\times \pi r^2$ + 150 $\times 2\pi r^2$

= 16.8 π + 150 πr^2

= 71. 68 cm^2

Ans: - 17 Let r_2 & r_2 be the two ends of the frustum $\frac{r_1}{20}$ = tan 30

$$r_1 = \frac{20}{\sqrt{3}}; r_2 \frac{10}{\sqrt{3}} cm$$

Volume of frustum = $\frac{1}{3}\pi$ h ($r^2{}_1 + r^2{}_2 + r_1 r_2$)

= $\frac{1}{3}\pi \times 10\left(\frac{400}{3}+\frac{100}{3}+\frac{200}{3}\right)$cm

Since the frustum is drawn into a wire of length x

Volume of frustum = volume of cylinder

$\frac{1}{3}\pi \times 10 \times \frac{700}{3} = \pi \left(\frac{1}{32}\right)^2 \times x$

⇨ x = <u>7168000</u> cm

9

x = 7964.4m

Ans: - 18 Self practice

Ans: - 19 Self practice

Ans: - 20 Volume of liquid in the vessel = $\frac{1}{3}\pi\ (7)^2\,(9) + \pi\,(7)^2\,(3)$

= 924 cu cm

Height of cylindrical part = $\dfrac{924}{\frac{22}{7}\times 49}$ = 6 cm

CHAPTER-14 : STATISTICS

- **Mean, median and mode of grouped data (bimodal situation to be avoided).**
- **Cumulative frequency graph.**

IMPORTANT POINTS TO REMEMBER

Measure of central Tendency

1. **Mean of Grouped Data**
   - **(i)** To find the mean of grouped data, it is assumed that the frequency of each class interval is centred around its mid-point
   - **(ii)** Direct Method
     Mean $(\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f i}$ where the x_i (class mark) is the mid-point of the i^{th} class interval and f_i is the corresponding frequency.
   - **(iii)** Assumed Mean Method
     Mean $(\bar{x}) = a + \frac{\Sigma f_i d_i}{\Sigma f i}$ where a is the assumed mean and $d_i = x_i - a$ are the deviations of x_i from a for each i
   - **(iv)** Step-deviation Method
     Mean $(\bar{x}) = a + h\,\frac{\Sigma f_i u_i}{\Sigma f i}$ where a is the assumed mean, h is the class size and $u_i = \frac{x_i - a}{h}$
   - **(v)** If the class sizes are unequal, the formula in (iv) can still be applied by taking h to be a suitable divisor of all the d_i's.
2. **Mode of Grouped Data**
   - **(i)** In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class is known as the modal class. The mode of the data is a value inside the modal class.
   - **(ii)** Mode of the grouped data can be calculated by using the formula
     Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$, where l is the lower limit of the modal class, h is the size of the class, f_1 is frequency of the modal class and f_0 and f_2 are the frequencies of the classes preceding and succeeding the modal class, respectively.
3. **Median of Grouped Data**
   (i) Cumulative frequency table – the less than type and the more than type of the grouped frequency distribution.
   (ii) If n is the total number of observations, locate the class whose cumulative frequency is greater than (and nearest to) $\frac{n}{2}$. This class is called the median class.
   (iii) Median of the grouped data can be calculated by using the formula
   Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] h$, where l is the lower limit of the median class, n is the number of observations, h is the class size, cf is the cumulative frequency of the class preceding the median class and f is the frequency of the median class.
4. **Graphical Representation of Cumulative Frequency Distribution (Ogive) – Less than type and more than type.**
   (i) To find median from the graph of cumulative frequency distribution (less than type) of a grouped data.
   (ii) To find median from the graphs of cumulative frequency distributions (of

less than type and more than type) as the abscissa of the point of intersection of the graphs.

IMPORTANT QUESTIONS FROM CHAPTER

1. While computing mean of grouped data, we assume that the frequencies are
   (A) evenly distributed over all the classes
   (B) centred at the classmarks of the classes
   (C) centred at the upper limits of the classes
   (D) centred at the lower limits of the classes

2. The arithmetic mean of 1,2,3,.............,n is

   A) $\frac{n+1}{2}$ B) $\frac{n}{2}$ C) $\frac{n-1}{2}$ D) $\frac{n}{2}+1$

3. Consider the following distribution:

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is
(A) 3 (B) 4 (C) 48 (D) 51

4. In the formula $\bar{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$, for finding the mean of a grouped frequency distribution, $u_i =$

(A) $\frac{x_i + a}{h}$ (B) $h(x_i - a)$ C) $\frac{x_i - a}{h}$ (D) $\frac{a - x_i}{h}$

5. Find the missing frequencies and the median for the following distribution if the mean is 1.46

No of accidents:	0	1	2	3	4	5	Total
Frequencies (no of days):	46	?	?	25	10	5	200

6. For the following distribution :

Marks	**Number of students**
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

The modal class is

(A) 10-20 (B) 20-30 (C) 30-40 (D) 50-60

7. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	No of leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Find the median length of leaves.

8. Calculate the mean for the following distribution:

x:	5	6	7	9	9
f:	4	8	14	11	3

9. If the mean of the following data is 20.6. Find the value of p.

x:	10	15	P	25	35
f:	3	10	25	7	5

10. If the mean of the following data is 15, find p

x:	5	10	15	20	25
f:	6	p	6	10	5

11. Find the value of p for the following distribution whose mean is 16.6

x:	8	12	15	p	20	25	30
f:	12	16	20	24	16	8	4

12. Find the missing value of p for the following distribution whose mean is 12.58

x:	5	8	10	12	p	20	25
f:	2	5	8	22	7	4	2

13. Find the missing frequency (p) for the following distribution whose mean is 7.68.

x:	3	5	7	9	11	13
f:	6	8	15	p	8	4

14. Compare the modal ages of two groups of students appearing for an entrance test:

Age in years	16 – 18	18 – 20	20 – 22	22 – 24	24 – 26
Group A	50	78	46	28	23
Group B	54	89	40	25	17

15. The following table shows the ages of the patients admitted in a hospital during a year:

Ages (in years):	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65
No of students:	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

16. Find missing frequency f if mode of given data is 154.

Class	120-130	130-140	140-150	150-160	160-170	170-180
Frequency	2	8	12	f	8	7

17. The following is the distribution of height of students of a certain class in a city:

Height (in cm):	160 – 162	163 – 165	166 – 168	169 – 171	172 – 174
No of students:	15	118	142	127	18

Find the average height of maximum number of students.

ANSWERS

1 (C)
2 (A)
3 (A)
4 (C)
5

No of accidents (x)	No of days (f)	fx
0	46	0
1	x	x
2	y	2y
3	25	75
4	10	40
5	5	25
	N = 200	Sum = x + 2y + 140

Given N = 200 46 + x + y + 25 + 10 + 5 = 200

x + y = 200 – 46 – 25 – 10 – 5 x + y = 114 ---- (1)

And, Mean = 1.46 Sum/ N = 1.46 (x + 2y + 140)/ 200 = 1.46 x + 2y

= 292 – 140 x + 2y = 152 ---- (2)

Subtract equation (1) from equation (2), we get

x + 2y – x – y = 152 – 114, y = 38

Putting the value of y in equation (1), we have x = 114 – 38 = 76

6 (C)

7 The given data is not having continuous class intervals. We can observe the difference between two class intervals is 1. So we have to add and subtract 1/2 = 0.5 to upper class limits and lower class limits Now continuous class intervals with respective cumulative frequencies can be represented as below:

Length (in mm)	Number of leaves f_i	Cumulative frequency (cf)
117.5 – 126.5	3	3
126.5 – 135.5	5	8
135.5 – 144.5	9	17
144.5 – 153.5	12	29
153.5 – 162.5	5	34
162.5 – 171.5	4	38
171.5 – 180.5	2	40

From the table we may observe that cumulative frequency just greater then n/2

(i.e. 40/2 = 20) is 29, belongs to class interval 144.5 – 153.5

Median class = 144.5 – 153.5 Lower limit (l) = 144.5 Class size (h) = 9 Frequency (f) of median class = 12 Cumulative frequency (c f) of class preceding median class = 17

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 144.5 + \left(\frac{20 - 17}{12}\right) \times 9)$$

= 144.5 + 9/4

= 146.75

So median length of leaves is 146.75 mm

8

x	f	fx
5	4	20
6	8	48
7	14	98
8	11	88
9	3	27
	N = 40	281

Mean = 281/40 =7.025

9

x	f	fx
10	3	30
15	10	150
P	25	25p
25	7	175
35	5	175
	N = 50	Sum = 2620 + 25P

Given Mean = 20.6

20.6 = (530 + 25p)/50

20.6 25p = 20.6 P
P = 20

10

x	f	fx
5	6	30
10	P	10p
15	6	90
20	10	200
25	5	125
	N = p + 27	Sum = 10p + 445

Given Mean =15

$$15 = \frac{10P + 445}{P + 127}$$

15P + 405 = 10P + 445

15P – 10P = 445-405

5P = 40
P = 8

11

x	f	fx
8	12	96
12	16	192
15	20	300
P	24	24p
20	16	320
25	8	200
30	4	120
	N = 100	Sum = 24p + 1228

Given Mean = 16.6

16.6 = (24p +1228)/100

1660 = 24p + 1228

24p = 1660 – 1228

P = 432/24

P = 18

12

x	f	fx
5	2	10
8	5	40
10	8	80
12	22	264
P	7	7p
20	4	80
25	2	50
	N = 50	Sum = 524 + 7p

Given mean = 12.58
Mean = Sum/N
12.58 = (524 +7p)/50
629 = 524 + 7p
629 - 524 = 7p
105 = 7P
P = 105/7
P = 15

13

x	f	fx
3	6	18
5	8	40
7	15	105
9	p	9p
11	8	88
13	4	52
	N = P + 41	Sum = 9p + 303

Given Mean = 7.68

$$7.68 = \frac{9P + 303}{P + 41}$$

7.68 (P + 41) = 9P + 303
7.68 P + 314.88 = 9P + 303
9P – 7.68 P = 314.88 – 303
1.32 P = 11.88

P = 11.88/1.32

P = 9

14

Age in years	16 – 18	18 – 20	20 – 22	22 – 24	24 – 26
Group A	50	78	46	28	23
Group B	54	89	40	25	17

For Group A Here the maximum frequency is 78, then the corresponding class 18 – 20 is model class l = 18, h = 20 - 18 = 2, f = 78, f_1 = 50, f_2 = 46

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 18 + \frac{78 - 50}{2 \times 78 - 50 - 46} \times 2$$

= 18 + 56/60

= 18 + 0.93

= 18.93 years

For group B Here the maximum frequency is 89, then the corresponding class 18 - 20 is the modal class l = 18, h = 20 - 18 = 2, f = 89, f_1 = 54, f_2 = 40 Mode

$$= l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 18 + \frac{89 - 54}{2 \times 89 - 54 - 40} \times 2$$

= 18 + 70/84

= 18 + 0.83

= 18.83 years

Hence the modal age for the Group A is higher than that for Group B

15 We may compute class marks (x_i) as per the relation

$$x_i = \frac{\text{Upper Class Limit} + \text{Lower Class Limit}}{2}$$

Now taking 30 as assumed mean (a) we may calculate d_i and f_id_i as follows:

Age (in years)	Number of patients f_i	Class marks x_i	$d_i = x_i - 275$	f_id_i
5 – 15	6	10	- 20	-120
15 – 25	11	20	- 10	-110
25 – 35	21	30	0	0
35 – 45	23	40	10	230
45 - 55	14	50	20	280
55 – 65	5	60	30	150
Total	80			430

From the table we may observe that $\Sigma f_i = 80$, $\Sigma f_i d_i = 430$,

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

= 30 + 430/80

= 30 + 5.375 = 35.375 = 35.38

Clearly, mean of this data is 35.38. It represents that on an average the age of a patients admitted to hospital was 35.38 years.

As we may observe that maximum class frequency is 23 belonging to class interval 35 – 45

So, modal class = 35 – 45 Lower limit (l) of modal class = 35 Frequency (f) of modal class = 23

Class size (h) = 10 Frequency (f_1) of class preceding the modal class = 21 Frequency (f_2) of class succeeding the modal class = 14 Mode

$$= l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 35 + \frac{23 - 21}{2 \times 23 - 21 - 14} \times 10$$

$$= 35 + \frac{2}{46 - 35} \times 10$$

$$= 35 + 1.81 = 36.8$$

Clearly mode is 36.8. It represents that maximum number of patients admitted in hospital were of 36.8 years.

16 Given, Mode = 154, so modal class is 150-160.

We know that, Mode = $L + \frac{f_1 - f_0}{2f_2 - f_1 - f_0} \times h$

Where, L = 150, f_0= 12, f_1= f, f_2= 8, h= 10

$154 = 150 + \frac{f-12}{2\times 8 - f - 12} \times h$

$\Rightarrow$ 154-150= $\frac{f-12}{2f-20}$

$\Rightarrow$ 4 = $\frac{f-12}{2f-20}$

$\Rightarrow$ By simplifying we get,

$f = 20$

17

Heights(exclusive)	160 – 162	163 – 165	166 – 168	169 – 171	172 – 174
Heights (inclusive)	159.5 – 162.5	162.5 – 165.5	165.5 – 168.5	168.5 – 171.5	171.5 – 174.5
No of students	15	118	142	127	18

Here the maximum frequency is 142, then the corresponding class 165.5 – 168.5 is the modal class

$l = 165.5$, $h = 168.5 - 165.5 = 3$, $f = 142$, $f_1 = 118$, $f_2 = 127$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 165.5 + \frac{142 - 118}{2 \times 142 - 118 - 127} \times 3$$

$= 165.5 + 72/39$

$= 165.5 + 1.85$

$= 167.35$ cm

CHAPTER-15 : PROBABILITY

- **Classical definition of probability.**
- **Simple problems on finding the probability of an event.**

IMPORTANT POINTS TO REMEMBER

- **PROBABILITY**

  The theoretical probability (also called classical probability) of an event A, written as P(A), is defined as

  $$P(A) = \frac{Number\ of\ outcomes\ favourable\ to\ A}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

- **COMPLIMENTARY EVENTS AND PROBABILITY**

  We denote the event 'not E' by E´ . This is called the complement event of event E.

  So, P(E) + P(not E) = 1

  i.e., P(E) + P(E) = 1, which gives us P(E) = 1 –P(E).

- **The probability of an event which is impossible to occur is 0. Such an event is called an impossible event.**
- **The probability of an event which is sure (or certain) to occur is 1. Such an event is called a sure event or a certain event.**
- **The probability of an event E is a number P(E) such that $0 \leq P(E) \leq 1$**
- **An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.**
- **DECK OF CARDS AND PROBABILITY**

  A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. They are black spades (♠) red hearts (♥), red diamonds (♦) and black clubs (♣).

  The cards in each suit are Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, Queens and Jacks are called face cards.

IMPORTANT QUESTIONS FROM CHAPTER

Q 1 Ranveer goes to school either by bus or uses his bicycle. Probability that he will use bus is 3/7. What is the probability that he will use his bicycle for going to school?

Q 2 A carton consist of 200 shirts of which 176 are good,16 have minor defects and 8 have major defects, A trader Jyoti will only accept two shirts which are good while another trader Riya will only reject the shirt which have major defects. One shirt is drawn at random from the carton. what is the probability that it is acceptable to
(a) Jyoti (b) Riya

Q 3 Cards marked with the numbers 3 to 102 are placed in a box and mixed thoroughly One card is drawn from the box. Find that the number on the card is a number less than 15, and a number which is perfect square.

Q 4 Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

Q 5 A coin is tossed twice. If the second throw results in tail, a die is thrown. Describe the sample space.

Q 6 What is the probability of having 53 Sundays in a leap year?

Q 7 Two dice are thrown at the same time and the probability that the difference of numbers appearing on the two dice is 2.

Q 8 Rita and Gita are friends .what is the probability that in a non leap year both will have
(i) different birthdays? (ii) Same birthdays?

Q 9 A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither a black card nor a queen.

Q 10 A bag containing 18 balls out of which x balls are red
(i) If one ball is drawn at random from the bag, then what is the probability that it is a red ball?
(ii) If 2 more red balls are put in the bag, then the probability of drawing a red ball will be 9/8 times that of probability of red ball in part (i). Find the value of x.

Q 11 A jar contains 54 marbles each of which is blue, green or white. The probability of selecting a blue marble at random from the jar is 1/3, and the probability of selecting a green marble at random is 4/9.How many white marbles does the jar contain?

Q 12 Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3, respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting

each sum from 2 to 9 separately.

Q 13 Cards with numbers 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has

(i) an even number (ii) a square number

Q 14 A die has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recorded.

(i) How many different scores are possible?

(ii) What is the probability of getting a total of 7?

Q 15 At a fete, cards bearing numbers 1 to 1000, one number on one card, are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square greater than 500, the player wins a prize. What is the probability that

(i) the first player wins a prize?

(ii) the second player wins a prize, if the first has won?

Q 16 In a game, the entry fee is Rs 5. The game consists of a tossing a coin 3 times. If one or two heads show, Sweta gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. For tossing a coin three times, find the probability that she

(i) loses the entry fee.

(ii) gets double entry fee.

(iii) just gets her entry fee.

Q 17 All the jacks, queens and kings are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 similar value for other cards, find the probability that the card has a value

(i) 7 (ii) greater than 7 (iii) less than 7

Q 18 Box A contains 25 slips of which 19 are marked Re 1and other are marked Rs 5 each. Box B contains 50 s lips of which 45 are marked Re 1each and others are marked Rs 13 each. Slips of both boxes are poured into a third box and reshuffled.

A slip is drawn at random. What is the probability that it is marked other than Re 1?

Q 19 A number x is selected from the numbers 1, 2, 3 and then a second number y is randomly selected from the numbers 1, 4, 9. Find the probability that the product xy of the two numbers will be less than 9.

Q 20 A coin is tossed two times. Find the probability of getting at most one head.

ANSWERS

1 Probability that Ranveer will go by bus = 3/7

probability that he will use his bicycle = 1 – 3/7 = 4/7

2 Total number of shirts = 200

Shirts acceptable to Jyoti = 176

Probability that it is acceptable to Jyoti = 176/200 =0.88

Probability that it is acceptable to Riya = 192/200 = 0.96

3 Probability that the card is a number less than 15 = 12/100

Probability that the card is a number which is perfect square = 9/ 100

4 Probability that the product is less than 9 = 16/ 36 = 4/9.

5 When a coin is tossed twice, the possible outcomes are [HH, HT, TH, TT]

The second throw results head i.e. HH,TH

The second throw results tail i.e. HT, TT

The possible outcomes on a die, i.e. 1, 2, 3,4,5,6.

Therefore sample space is

S={HH,TH,HT1,HT2,HT3,HT4,HT5,HT6,TT1,TT2,TT3,TT4, TT5, TT6}

6 leap year = 366 days = 52 weeks + 2 days

2 days could be = (M,T),(T,W),(W,TH),(TH,F),(F,SAT),(SAT,SUN),(SUN,M)

Total outcomes =7

Probability of having 53 Sundays in a leap year = 2/7

7 Probability that the difference of numbers appearing on the two dice is 2 = 8/36 = 2/9

8 i) Probability that both will have different birthdays = 364/365

ii) Probability that both will have same birthdays = 1/365.

9 Probability that the card drawn is neither a black card nor a queen = 24/52 = 6/13

10 (i) Total number of balls in the bag = 18

Total number of red balls in the bag = x

∴ P (getting a red ball) = $\frac{x}{18}$

(ii)Number of red balls added to the bag = 2

Total number of balls in the bag = 18 + 2 = 20

Total number of red balls in the bag = x + 2

∴ P (getting a red ball) = $\frac{x+2}{20}$

According to the question

$\frac{x+2}{20} = \frac{9}{8}\left(\frac{x}{18}\right)$, on simplification , x = 8

11 P(getting blue marble) = b/54 = 1/3

So, b = 18

P(getting green marble) = g/54 =4/9

So, g = 24

B + g + w = 54

18 + 24 + w = 54 ,

W = 12.

12 Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3 can be thrown in following ways.

(1, 1) (1, 1) (1, 2) (1, 2) (1, 3) (1, 3)

(2, 1) (2, 1) (2, 2) (2, 2) (2, 3) (2, 3)

(3, 1) (3, 1) (3, 2) (3, 2) (3, 3) (3, 3)

(4, 1) (4, 1) (4, 2) (4, 2) (4, 3) (4, 3)

(5, 1) (5, 1) (5, 2) (5, 2) (5, 3) (5, 3)

(6, 1) (6, 1) (6, 2) (6, 2) (6, 3) (6, 3)

Favourable ways of getiing a sum 2 as (1, 1) (1, 1)

Required probability = $\frac{2}{36} = \frac{1}{18}$

Favourable ways of getiing a sum 3 as (1, 2) (1, 2) (2, 1) (2, 1)

Required probability = $\frac{4}{36} = \frac{1}{9}$

Favourable ways of getiing a sum 4 as (1, 3) (1, 3) (2, 2) (2, 2) (3, 1) (3, 1)

Required probability = $\frac{6}{36} = \frac{1}{6}$

Favourable ways of getiing a sum 5 as (2, 3) (2, 3) (3, 2) (3, 2) (4, 1) (4, 1)

Required probability = $\frac{6}{36} = \frac{1}{6}$

Favourable ways of getiing a sum 6 as (3, 3) (3, 3) (4, 2) (4, 2) (5, 1) (5, 1)

Required probability = $\frac{6}{36} = \frac{1}{6}$

Favourable ways of getiing a sum 7 as (4, 3) (4, 3) (5, 2) (5, 2) (6, 1) (6, 1)

Required probability = $\frac{6}{36} = \frac{1}{6}$

Favourable ways of getiing a sum 8 as (5, 3) (5, 3) (6, 2) (6, 2)

Required probability = $\frac{4}{36} = \frac{1}{9}$

Favourable ways of getiing a sum 9 as (6, 3) (6, 3)

Required probability = $\frac{2}{36} = \frac{1}{18}$

13 Total cards = 100

Even numbers are 2,4,6,8, …………98,100

Favourable cases = 50

So P(an even numbers) = $\frac{50}{100} = \frac{1}{2}$

Squared numbers are 4,9,16,25………81,100

Favourable cases = 9

So P(a squared number) = $\frac{9}{100}$

14 (i) How many different scores are possible?

(ii) What is the probability of getting a total of 7?

Two dice are numbered 0, 1, 1, 1, 6, 6 and 0, 1, 1, 1, 6, 6 can be thrown in following ways.

(0, 0) (0, 1) (0, 1) (0, 1) (0, 6) (0, 6)

(1, 0) (1, 1) (1, 1) (1, 1) (1, 6) (1, 6)

(1, 0) (1, 1) (1, 1) (1, 1) (1, 6) (1, 6)

(1, 0) (1, 1) (1, 1) (1, 1) (1, 6) (1, 6)

(6, 0) (6, 1) (6, 1) (6, 1) (6, 6) (6, 6)

(6, 0) (6, 1) (6, 1) (6, 1) (6, 6) (6, 6)

Different scores which are possible are 0, 1, 2, 6, 7, 12

Favourable cases are 6

Required probability = $\frac{6}{36} = \frac{1}{6}$

Favourable ways of getting a sum 7 as (1, 6) (1, 6) (1, 6) (1, 6) (1, 6) (1, 6) (6, 1) (6, 1) (6, 1) (6, 1) (6, 1) (6, 1)

Required probability = $\frac{12}{36} = \frac{1}{3}$

15 (i) First player can select a card from a box = 1000.

Perfect square greater then 500 are 9 (529, 576, 625, 676, 729, 784, 841, 900, 961)

P (the first player wins a prize) = $\frac{9}{1000}$ = 0.009

(ii) P(the second player wins a prize, if the first has won) = $\frac{8}{999}$

16 A coin is tossed three times

S={HHT,HTH,THH,HTT,THT,TTH,HHH,TTT}

There are 8 possible outcomes .n(S) = 8

The game consists of tossing a coin 3 times.

If one or two heads show. Sweta gets her entry fee back.

If she throws three heads, she receives double the entry fees.

If she gets TTT, she loses the entry fees.

(i) Loses the entry fee

Out of 8 possible outcomes, only one (TTT) is favourable.

$\therefore$ P(loses the entry fee) = $\frac{1}{8}$

(ii) Gets double entry fee

Out of 8 possible outcomes, only one (HHH) is favourable.

$\therefore$ P (Gets double entry fee) = $\frac{1}{8}$

(iii) Let E be event that she just gets her entry fees. Then

E = {HHT,HTH,THH,HTT,THT,TTH}

n(E) = 6

P(just gets her entry fees) = $\frac{n(E)}{n(s)} = \frac{6}{8} = \frac{3}{4}$

17 (i) Probability that the card has a value 7 = $\frac{4}{40} = \frac{1}{10}$

(ii) Probability that the card has a value greater than 7 = $\frac{12}{40} = \frac{3}{10}$

(iii) Probability that the card has a value less than 7 = $\frac{24}{40} = \frac{3}{5}$

18 Number of slips which are poured into a third box 25 + 50 = 75

Number of slips which are marked other than Rs 1 = (25 - 19) + (50 - 45) = 11

Probability that it is marked other than Re 1 = $\frac{11}{75}$

19 As a number x is selected from the numbers 1, 2, 3 and then a second number y is randomly selected from the numbers 1, 4, 9.

Total cases are 9 because of (1,1) (1,4) (1,9) (2,1) (2,4) (2,9) (3,1) (3,4) (3,9)

Favourable cases are 5 because of (1,1) (1,4) (2,1) (2,4) (3,1)

Required probability = $\frac{5}{9}$

20 When a coin is tossed twice, the possible outcomes are [HH, HT, TH, TT]$\Rightarrow$

$n(S) = 4$

Getting at most one head = [TT, HT, TH] $\Rightarrow n(E) = 3$

So P(E) = $\frac{3}{4}$